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I-501
09/13/2017

Emergence of scaling in random networks
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Key points

- One of the most cited papers of all time!
- A lot of systems around us are complex networks, such as genetic networks, nervous system, social networks, and World Wide Web.
- First model on networks: Random graph theory by Erdös and Rényi (ER)
- High degree of self-organization in complex networks is reported: The probability of a node having degree *k*, denoted P(k), follows a power law scaling, i.e. $P(k)=k^{-\gamma}$.
- Two key features are shown to cause this scaling: growth and preferential attachment. Existing network models fail to include these features.
- Real-life networks have the power-law scaling in their degree distributions: Actor collaboration network ($\gamma=2.3$), World Wide Web ($\gamma=2.1$), electrical power grid of western US ($\gamma=4$), citation patterns of publications ($\gamma=3$).
- Existing network models have different degree distributions. ER has a Poisson distribution. The small-world model proposed by Watts and Strogatz (WS) has a degree distribution similar to ER's (it is not Poisson though). So, in both models, it is practically impossible to find high degree nodes, which is not the case in real-life networks.
- In ER and WS models, number of nodes are fixed, so no growth in networks. Also, connections (edges) between nodes are made arbitrarily. However, networks both grow and the connections are made following preferential attachment!
- The model: growth + preferential attachment = scale-free
- Start with m_0 vertices, add a vertex on at a time, and connect it to *m* nodes following the preferential attachment rule. At a big enough t, where the network has m_0+t vertices and *mt* edges, power law is observed in degree distribution with $\gamma=2.9$.
- Verification: Both growth and preferential attachment are necessary in the model:
    - No preferential attachment: No power law, $P(k)=e^{-\beta k}$
    - No growth: No power law, no stationary P(k)
- Preferential attachment: rich-get-richer
- After enough time: $P(k)=2m^2/k^3$, and $\gamma=3$ independent of m. So, the model proposed is limited to a single γ value.
- Modify the model to get different γ values, such as using non-linear preferential attachment, changing a fraction of links to directed, adding edges between existing nodes.
- Growth and preferential attachment are both common mechanisms in business networks, social networks, etc. These mechanisms can help in explaining social and economic systems, and even possibly evolution.

Questions

- Do the two features, growth and preferential attachment, exist in real-life networks? Do they, especially growth, work only one way?
- Do the networks only change through nodes? What about adding edges between existing nodes or removing existing edges?
- Is the model complete in the sense of its representation range of networks? Can it be improved in any way? What would be done?
- Are there any phenomenon appearing in real-life networks disregarded in this model? For example, are there any characteristics of ER or WS models that appear in real-life and not considered in this model?