Emergence of scaling in random networks

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- Barabási, Albert-László, and Réka Albert. "Emergence of scaling in random networks." science 286.5439 (1999): 509-512.
- Number of citations: 29239 (Google scholar)

- Complex networks: vertices - edges
  - Genetic network: Proteins & genes - chemical interactions
  - Nervous system: Nerve cells - axons
  - World Wide Web: HTML pages - links
- First model: Random graph theory by Erdös and Rényi

- Power law
    - Self-organize into a scale-free state
    - $P(k) = k^{-\gamma}$
- Two features cause this scaling
    - Growth
    - Preferential attachment
- Not present in existing network models

- Collaboration of movie actors (A)

$$\gamma = 2.3$$

- World Wide Web (B)

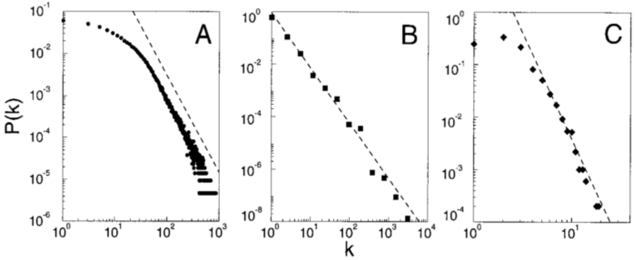
$$\gamma = 2.1$$

- Electrical power grid of the western US (C)

$$\gamma = 4$$

- Citation patterns

$$\gamma = 3$$



- Erdös-Rényi (ER) model (random graph)
- Watts-Strogatz (WS) model (small-world)
  - Probability of highly connected nodes decreases exponentially
  - No highly connected nodes
- In contrast with power law

Growth

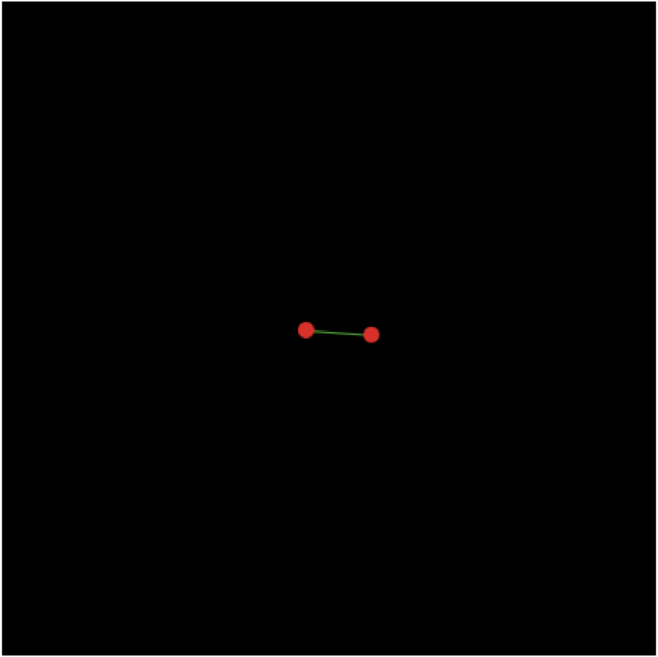
- ER and WS models are static: fixed number of nodes
- However...
  - Actors network: new actors join
  - WWW: new webpages are created
  - Citation network: New publication are done
- Networks expand!

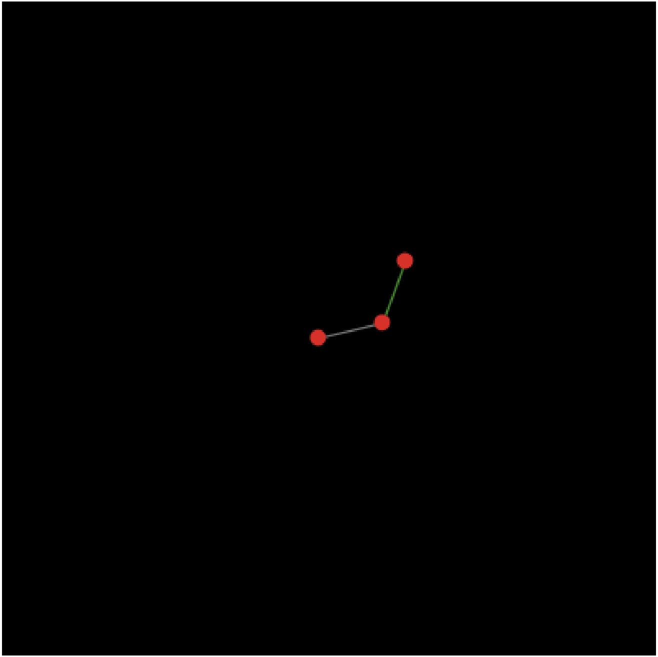
Preferential Attachment

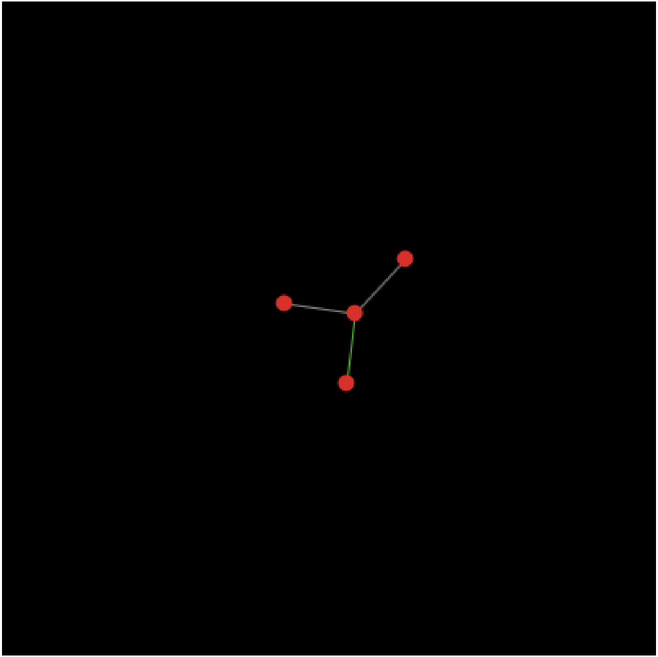
- The probability of an edge existing between any two nodes is the same
- However:
  - New actor more likely to be cast with an established actor
  - New webpage more likely to link to a popular webpage
  - New paper more likely to cite a well-known paper
- The connections formed by a vertex are not arbitrary!

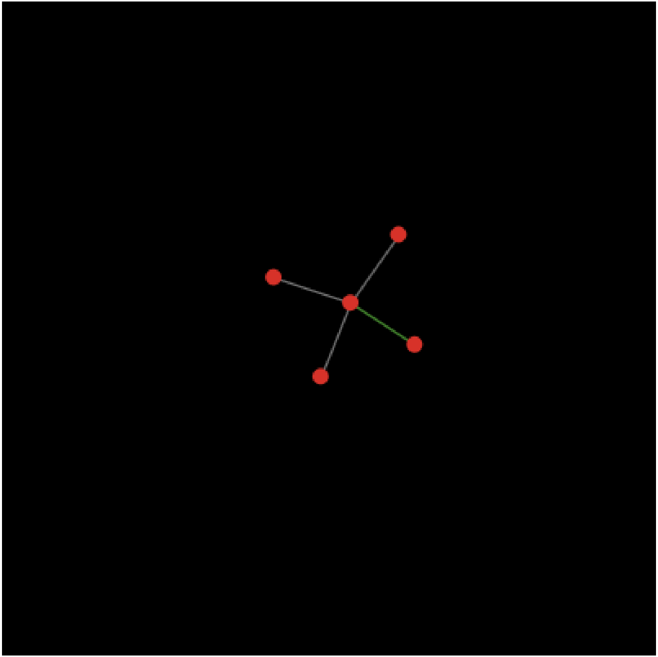
The Model

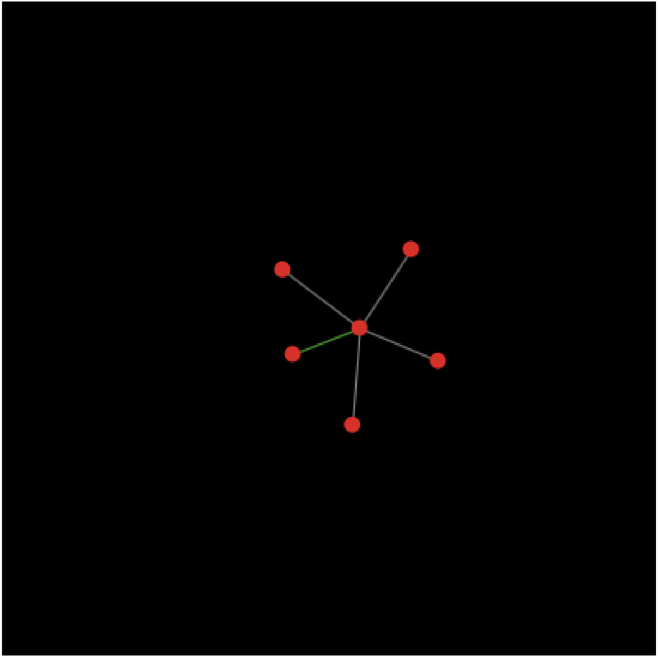
- Growth + preferential attachment = scale-free
- Growth: each new vertex connects to m already existing vertices
- Preferential attachment: $\pi(k_i) = \frac{k_i}{\sum_j k_j}$

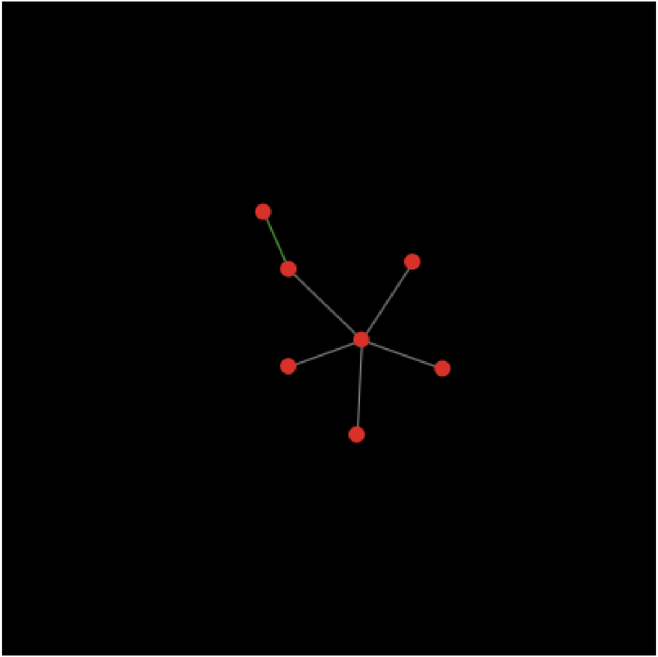


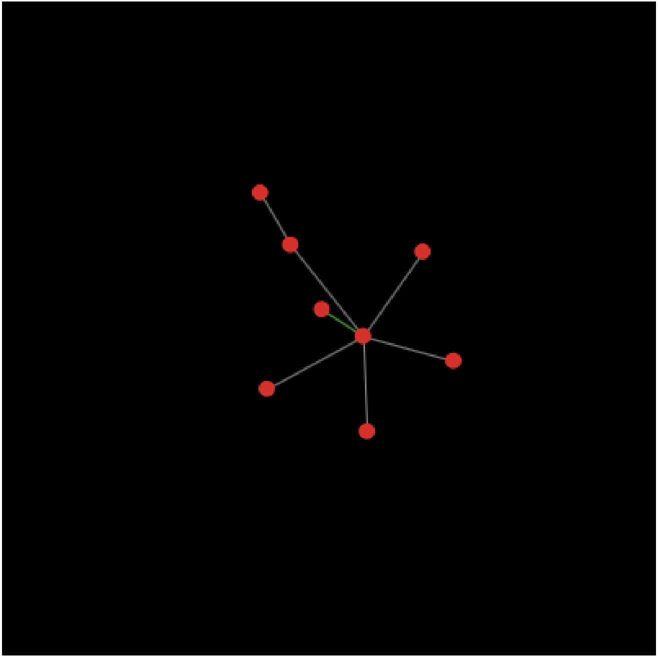


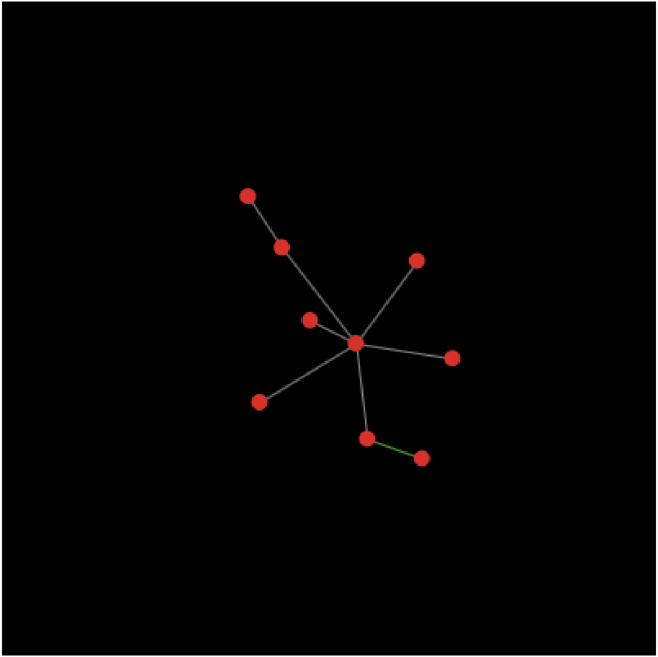


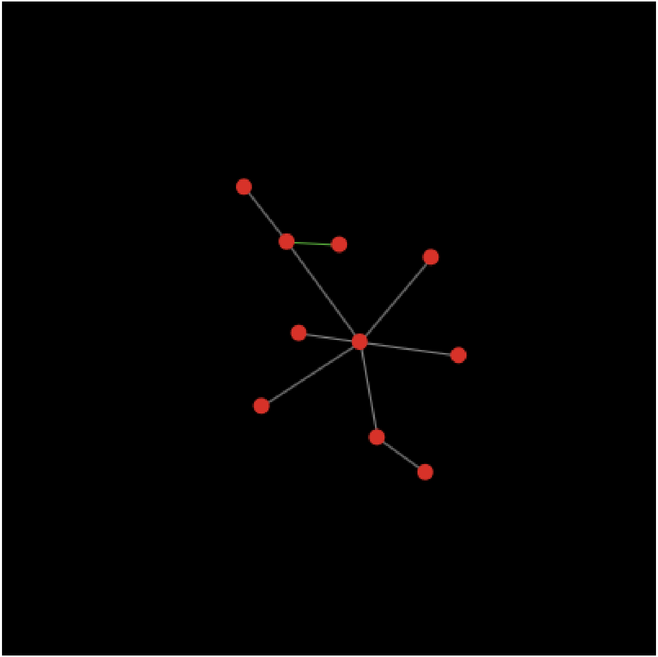












The Model

- Growth + preferential attachment = scale-free
- Growth: each new vertex connects to m already existing vertices
- Preferential attachment: $\pi(k_i) = \frac{k_i}{\sum_j k_j}$
- At time t:
  - $m_0 + t$ vertices
  - mt edges
  - $\gamma = 2.9 \pm 0.1$
- $P(k)$ independent of t

Verification

- No preferential attachment

$$P(k) = exp(-\beta k) \rightarrow \textit{no power-law}$$

- No growth

$$\textit{No stationary } P(k) \rightarrow \textit{no power-law}$$

- Preferential attachment: rich-get-richer
- After enough time:

$$P(k) = \frac{2m^2}{k^3}$$

- $\gamma = 3$, independent of m
  - Restricted to a single γ value
- To get different γ values:
  - Use non-linear preferential attachment
  - Change a fraction of links to directed links
  - Add edges between existing vertices

- Growth & preferential attachment: Common mechanisms in real life networks
- Even genetic networks may have these mechanisms
- Can be able to explain social and economic systems

Questions

- Are the two features, growth & preferential attachment, realistic for the networks in real life? Does growth work only as explained?
- Does the change in networks happen only through nodes?
- Is the model complete? Can it be improved in any way? What would be done?
- Are there any phenomenon appearing in networks disregarded in this model?