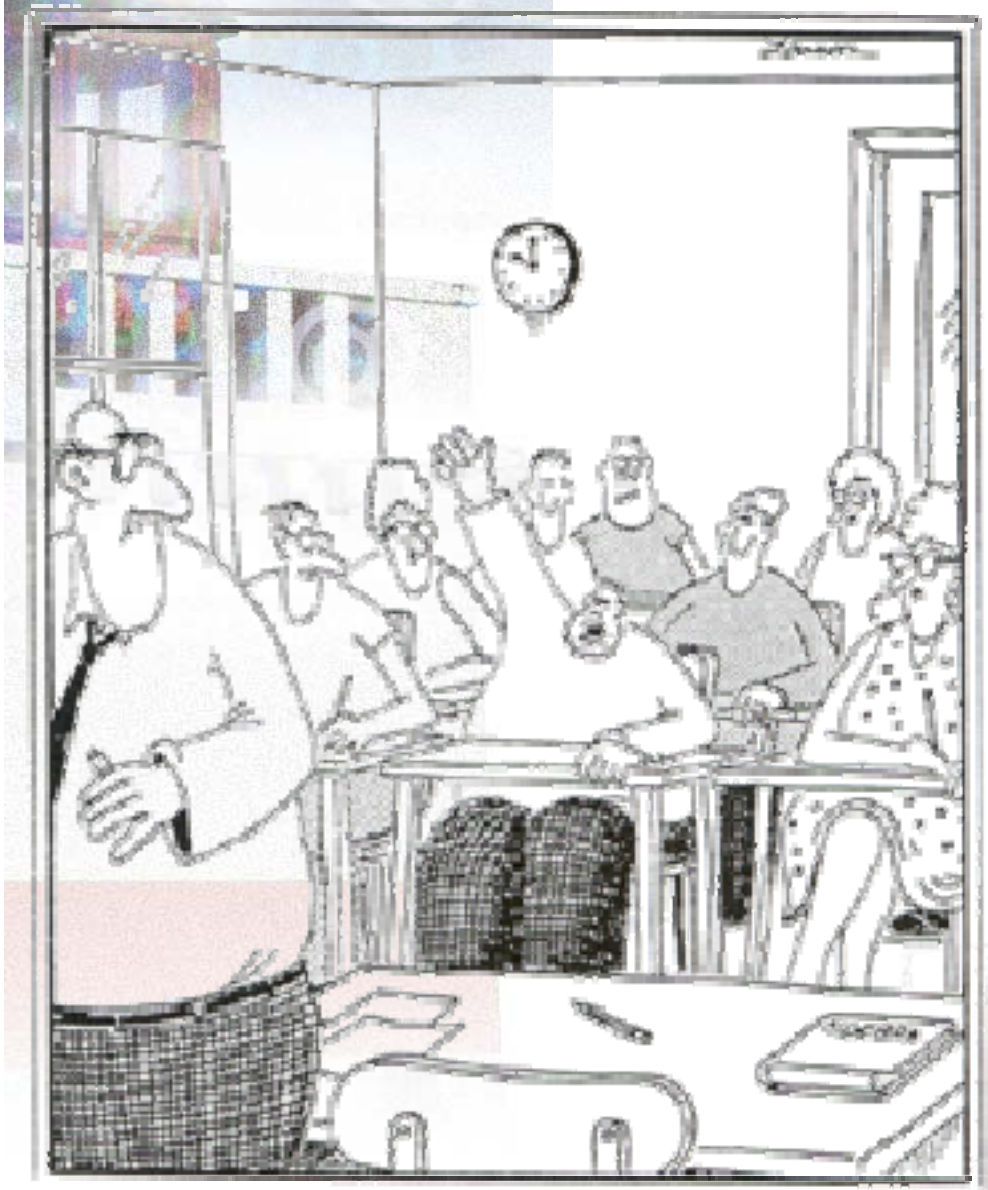
Introduction to Informatics



"Ms. Osborne, may I be excused? My brain is full."

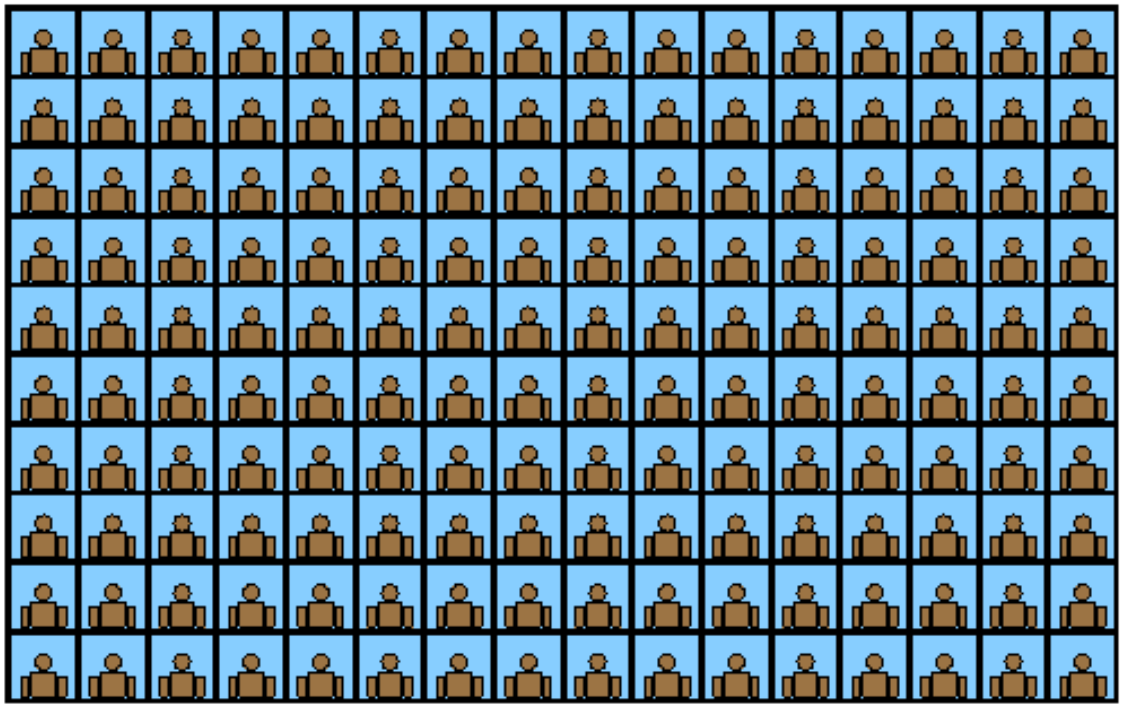
Lecture 13: Propositional and Boolean Logic

Luis M.Rocha and Santiago Schnell

Readings until now

- Lecture notes
  - Posted online
    - http://informatics.indiana.edu/rocha/i101
      - *The Nature of Information*
      - *Technology*
      - *Modeling the World*
  - @ *infoport*
    - *http://infoport.blogspot.com*
  - From course package
    - Von Baeyer, H.C. [2004]. *Information: The New Language of Science*. Harvard University Press.
      - Chapters 1, 4 (pages 1-12)
    - From Andy Clark's book "*Natural-Born Cyborgs*"
      - Chapters 2 and 6 (pages 19 - 67)
    - From Irv Englander's book "*The Architecture of Computer Hardware and Systems Software*"
      - Chapter 3: Data Formats (pp. 70-86)
    - Klir, J.G., U. St. Clair, and B.Yuan [1997]. Fuzzy Set Theory: foundations and Applications. Prentice Hall
      - Chapter 2: Classical Logic (pp. 87-98)

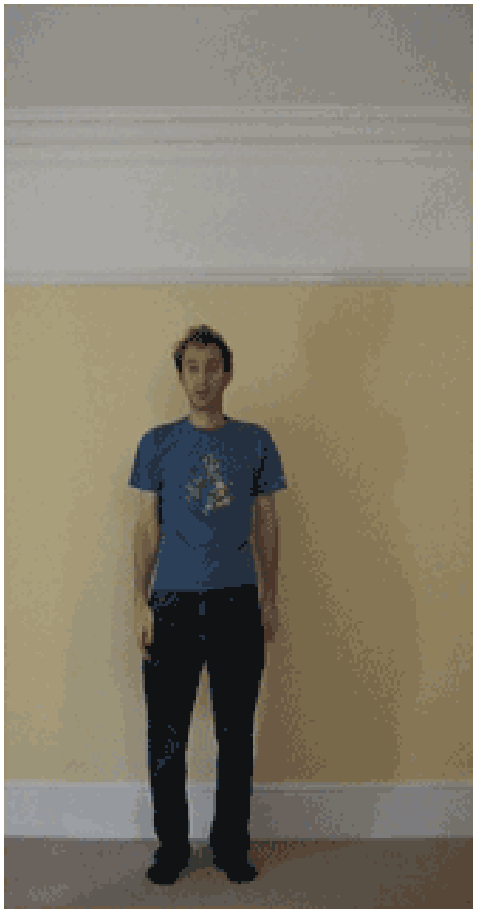
NO LAB THIS WEEK !!!



Luis M.Rocha and Santiago Schnell

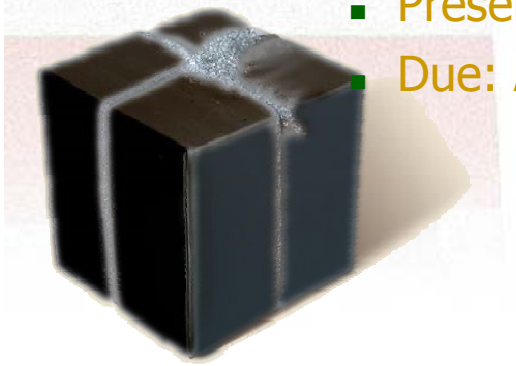
Assignment Situation

- Labs
  - Past
    - Lab 1: Blogs
      - Closed (Friday, January 19): Grades Posted
    - Lab 2: Basic HTML
      - Closed (Wednesday, January 31): Grades Posted
    - Lab 3: Advanced HTML: Cascading Style Sheets
      - Closed (Friday, February 2): Grades Posted
    - Lab 4: More HTML and CSS
      - Closed (Friday, February 9): Grades Posted
    - Lab 5: Introduction to Operating Systems: Unix
      - Closed (Friday, February 16): Being graded
    - Lab 6: More Unix and FTP
      - Due Friday, February 23
  - Next: Lab 7
    - Intro to Statistical Analysis using Excel
      - March 1 and 2, due Friday, March 9
- Assignments
  - Individual
    - First installment
      - Closed: February 9: Grades Posted
    - Second Installment
      - Due: March: 2nd
  - Group Project
    - First installment
      - Presented: March 6, Due: March 9th
- Midterm Exam
  - March 1st (Thursday)



Individual assignment

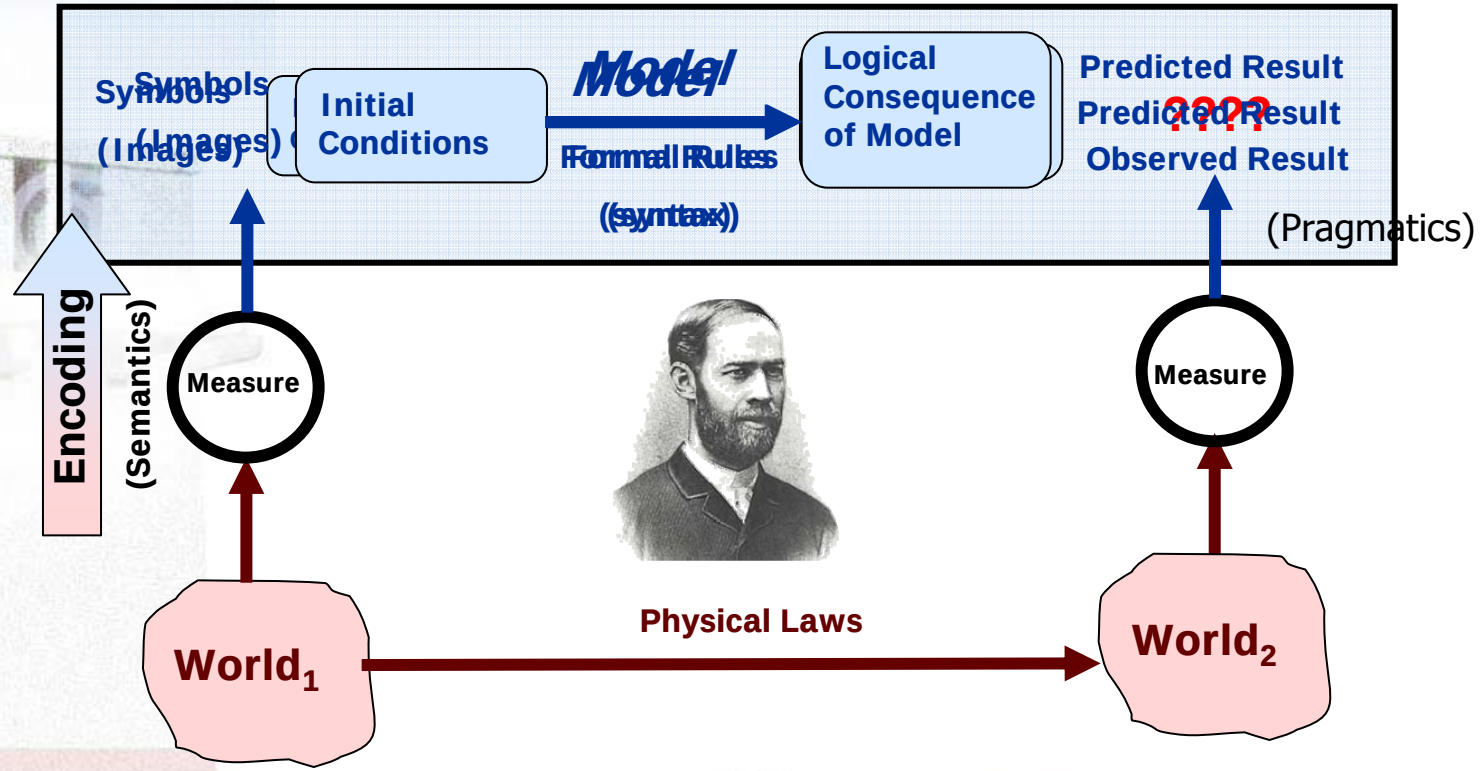
- Individual Project
  - 1st installment
    - Presented: February 1st
    - Due: February 9th
  - 2nd Installment
    - Presented: February 15th
    - Due: March: 2nd
  - 3rd Installment
    - Presented: March 8th
    - Due: March 30th
  - 4th Installment
    - Presented: April 5th
    - Due: April 20th



The Black Box

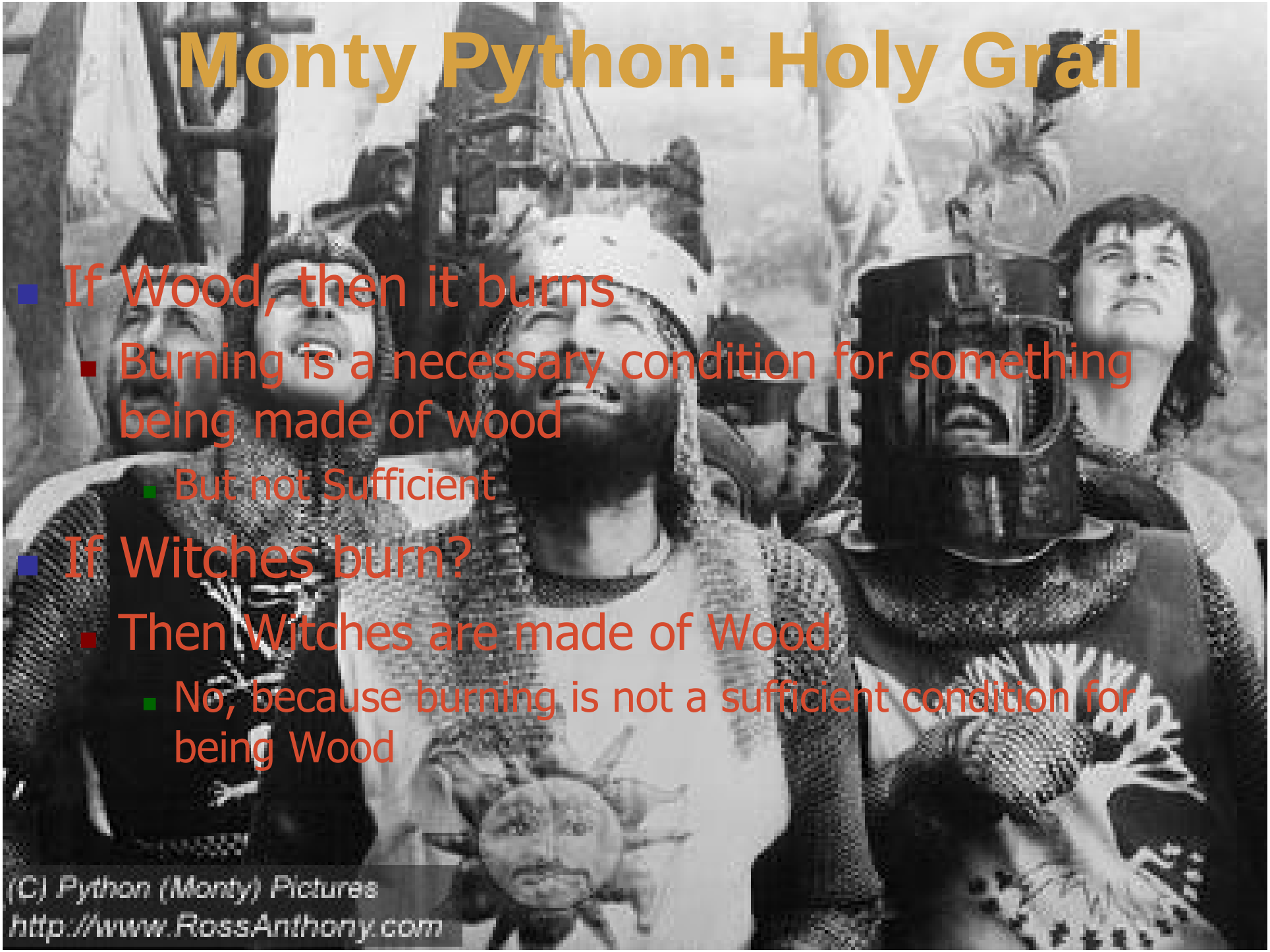
The Modeling Relation

Hertz' Modeling Paradigm



- Formal Rules
  - From symbolic representations of observables
  - Produce Conclusions

Monty Python: Holy Grail



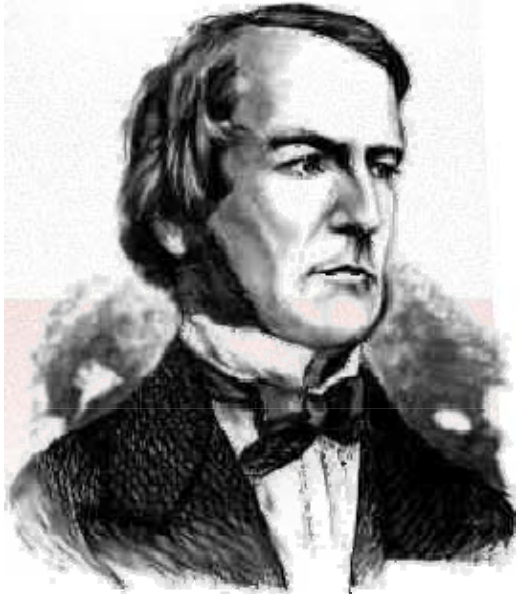
- If Wood, then it burns
  - Burning is a necessary condition for something being made of wood
    - But not Sufficient
- If Witches burn?
  - Then Witches are made of Wood
    - No, because burning is not a sufficient condition for being Wood

The structure of propositional logic

- *Simple propositions* are represented by single, lower case letters
  - Bloomington is a town – *p*
  - Indiana is a state - *q*
- *Complex propositions* are constructed by applying logical operations to simple propositions
  - Bloomington is a town *and* Indiana is a state – *p* and *q*
- *Logic Operations*
  - Conjunction [and] $\wedge$
  - Disjunction [or] $\vee$
  - Negation [not] $\neg$
  - Conditional [implies] $\Rightarrow$ (if, then)
  - Biconditional [equivalent] $\Leftrightarrow$ (if and only if)

Boolean Algebra/Logic

- The processing of information in Digital computers is based on Boolean Logic
  - Voltage used to represent (analog!) two states
    - False ⇔ 0
      - Lower Voltage ≈ [0,2] Volts
    - True ⇔ 1
      - Higher Voltage ≈ [2,5] Volts
  - Computers are physical devices to constrain signals to be in one range or the other
  - Logic Gate
    - Device that performs a basic logical operation on electric signals
      - One or more signals as input, producing one signal as output
    - Six important (three fundamental) operations
      - NOT: $\bar{A}$
      - AND: $A \bullet B$
      - OR: $A + B$
      - XOR: $A \oplus B$
      - NAND
      - NOR
  - Circuits
    - Combination of gates to compute more complicated tasks
  - Notational Method to describe gates and circuits
    - Boolean or Propositional Logic Expressions
    - Logic Diagrams
    - Truth Tables



George Boole (1815 – 1864)

Logic Operations/Gates: Negation/NOT

Suppose *p* is a proposition.

The *negation* of *p* is written $\neg p$ and it means:

"It is not the case that *p*."

Example - I101 is NOT Sarah's favorite class

Boolean Truth table for negation:

A	$X = \bar{A}$
0	1
1	0

$|\bar{A}| = 1 - A$

p	$\neg p$
F	T
T	F

Diagram

Adapted from C. Heeren

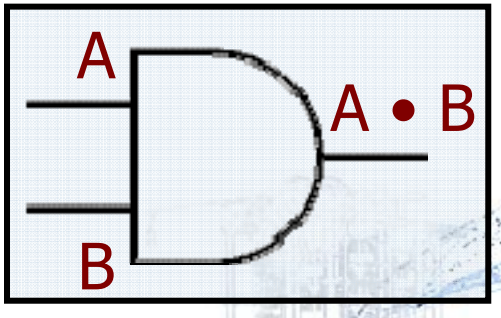
Conjunction/AND

- Conjunction: $p \wedge q$ corresponds to English "and."
- Proposition $p \wedge q$ is true when p and q are both true.

Example – Uma is blond and clever

Boolean Truth table for conjunction:

A	B	A·B
0	0	0
0	1	0
1	0	0
1	1	1



p	q	$p \wedge q$
F	F	F
F	T	F
T	F	F
T	T	T

$$|p \wedge q| = \min(|p|, |q|)$$

Adapted from C. Heeren

Disjunction/OR

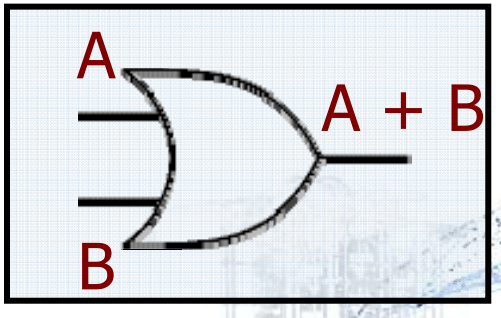
- Disjunction: $p \vee q$ corresponds to English "or."
- Proposition $p \vee q$ is true when p or q (or both) are true.

Example – Madonna is blond or clever

Boolean Truth table for disjunction:

p	q	$p \vee q$
F	F	F
F	T	T
T	F	T
T	T	T

A	B	A + B
0	0	0
0	1	1
1	0	1
1	1	1



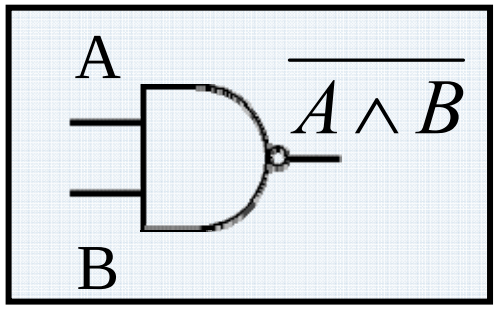
$$|p \vee q| = \max(|p|, |q|)$$

Adapted from C. Heeren

Other Boolean Gates

- NOT AND (NAND)

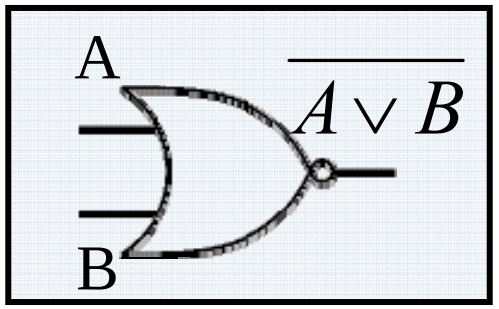
$$\overline{A \bullet B}$$



A	B	$\overline{A \wedge B}$
0	0	1
0	1	1
1	0	1
1	1	0

- NOT OR (NOR)

$$\overline{A + B}$$



A	B	$\overline{A \vee B}$
0	0	1
0	1	0
1	0	0
1	1	0

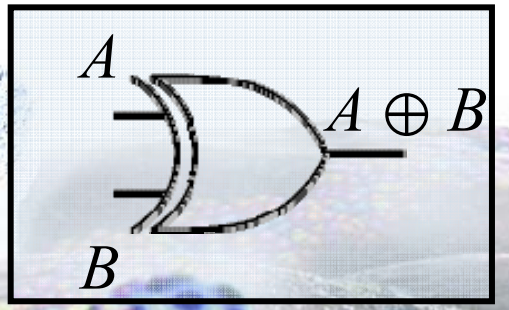
- Exclusive-OR (XOR): $\oplus$

A	B	$A \oplus B$
0	0	0
0	1	1
1	0	1
1	1	0

$$A \oplus B = (A + B) \bullet \left(\overline{A \bullet B}\right)$$

A or B and not both A and B

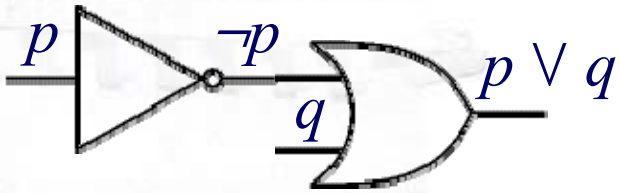
A or B and (A NAND B)



Implication or Conditional

Implication: $p \Rightarrow q$ corresponds to English "if…then…"
or "p only if q" or "p in case q"

Example: Bonnie will pass I101, only if she is awake in classes



$$p \Rightarrow q \equiv \neg p \vee q$$

Antecedent Consequent

Truth table for implication:

p	q	$p \Rightarrow q$
0	0	1
0	1	1
1	0	0
1	1	1

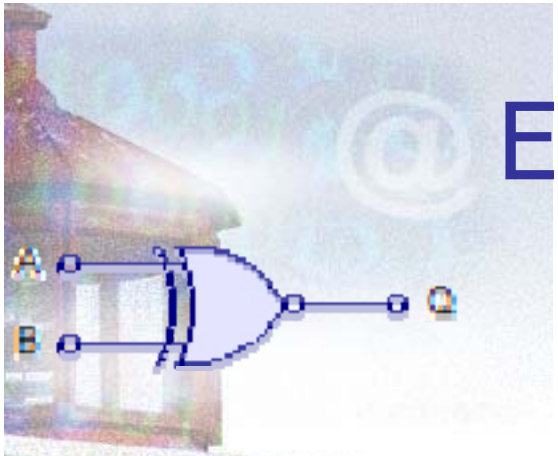
q is a ***necessary condition*** for p

p is a ***sufficient condition*** for q

Having a microscope (or some other instrument) is a **necessary condition** for (our) seeing viruses

If someone sees viruses, **then** that person uses a microscope

Adapted from C. Heeren



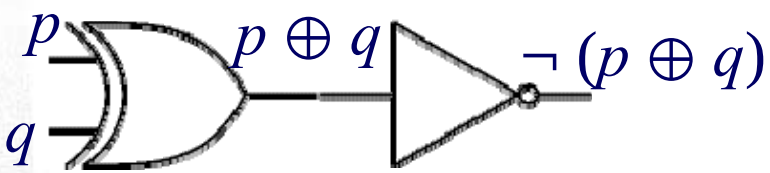
Equivalence or Biconditional

Equivalence: $p \Leftrightarrow q$ corresponds to English "if and only if…then…"

$(p \Rightarrow q) \wedge (q \Rightarrow p)$

Example – Justin will pass I101 if and only if he is awake in classes

$$p \Leftrightarrow q \equiv \neg(p \oplus q) \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$$



Antecedent

Consequent

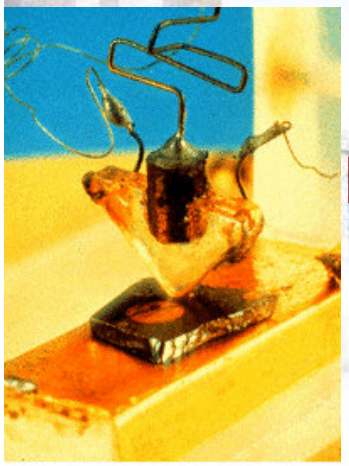
Truth table for equivalence:

p	q	$p \Leftrightarrow q$
0	0	1
0	1	0
1	0	0
1	1	1

Equivalence related to ***Necessary and Sufficient Condition***: q is a necessary and sufficient condition for p and p is a necessary and sufficient condition for q

Adapted from C. Heeren

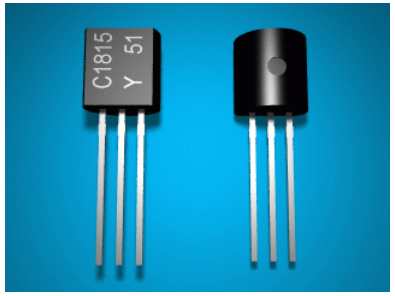
Semi-conductor Transistors

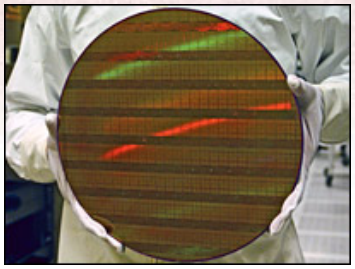


- Invented by John Bardeen, Walter Houser Brattain, and William Bradford Shockley
  - at Bell Laboratories in December 1947
  - awarded the Nobel Prize in physics in 1956.

Function as switches

- A device for making or breaking an electric circuit
  - Also for amplification in analog devices
- For choosing between several states
  - Between "on" and "off", "1" or "0"
  - Allows the construction of *logic gates*
- Semiconductor device
  - principally *silicon*, germanium and gallium arsenide.
- Better than vacuum tubes
  - Smaller size, Highly automated manufacture, Lower cost, Lower operating voltage, Absence of a heater Lower power dissipation etc.



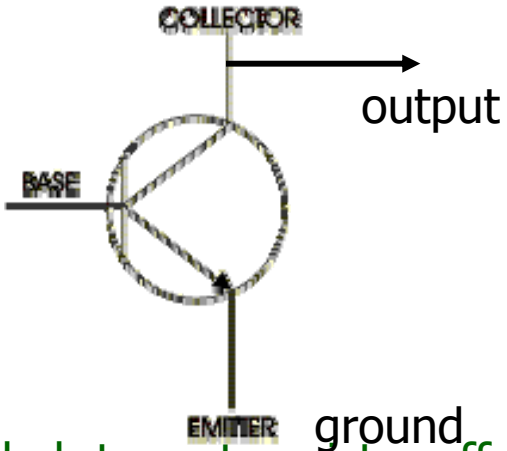


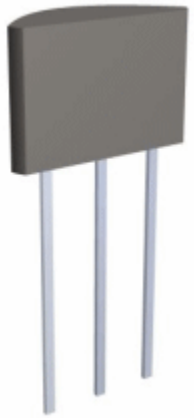
Smallest Transistor

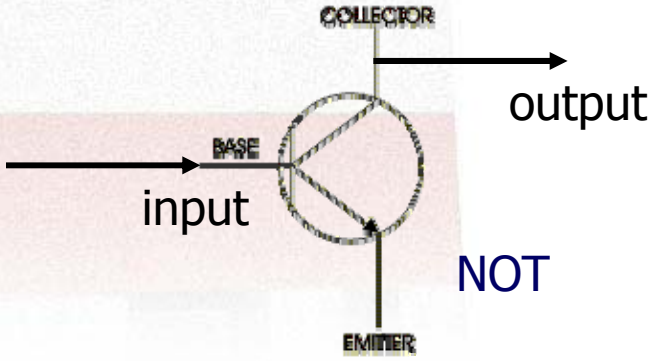
- Intel 2007: 45 nanometres (billionths of a metre) wide

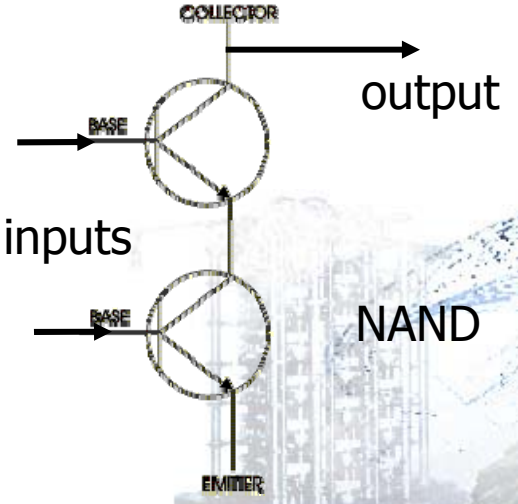
Implementing Logic Gates

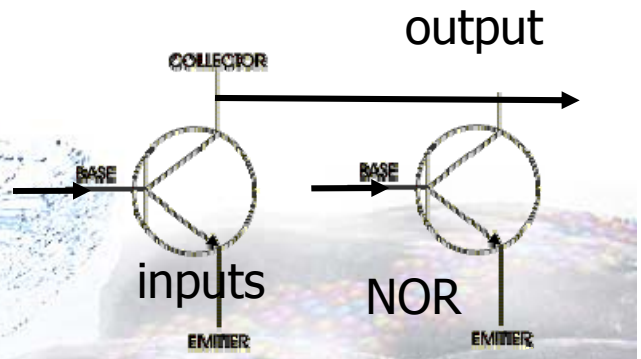
- Transistor
  - Source or collector
    - Produces high voltage
      - 5 volts
  - Base regulates
    - If signal high
      - Source signal gets grounded: turns transistor off (0)
    - If signal is low
      - Source signal stays high: transistor on (1)
  - NOT, NAND and NOR gates easiest to produce



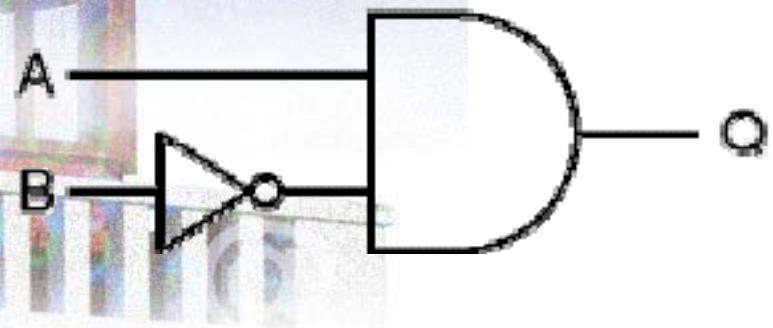




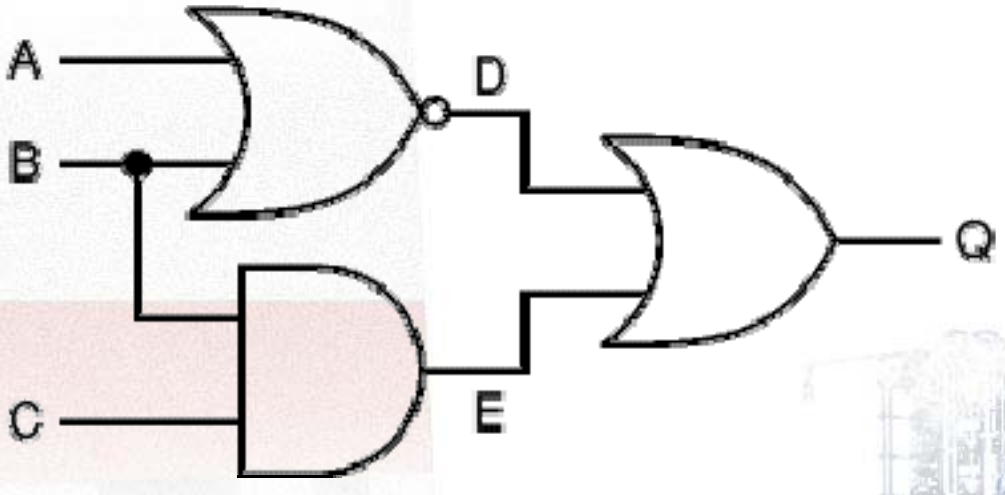




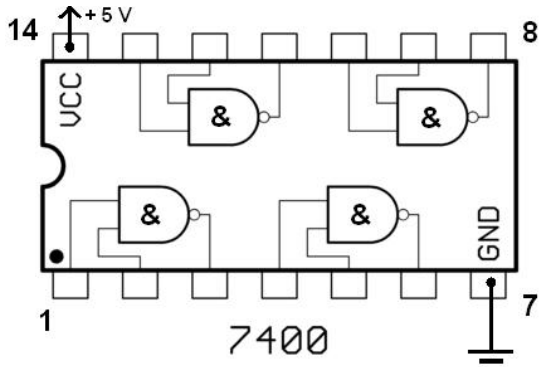
Combinational Circuits

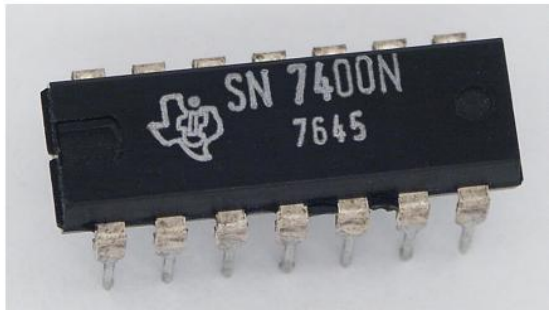


A AND NOT B



(NOT (A OR B)) OR (B AND C)





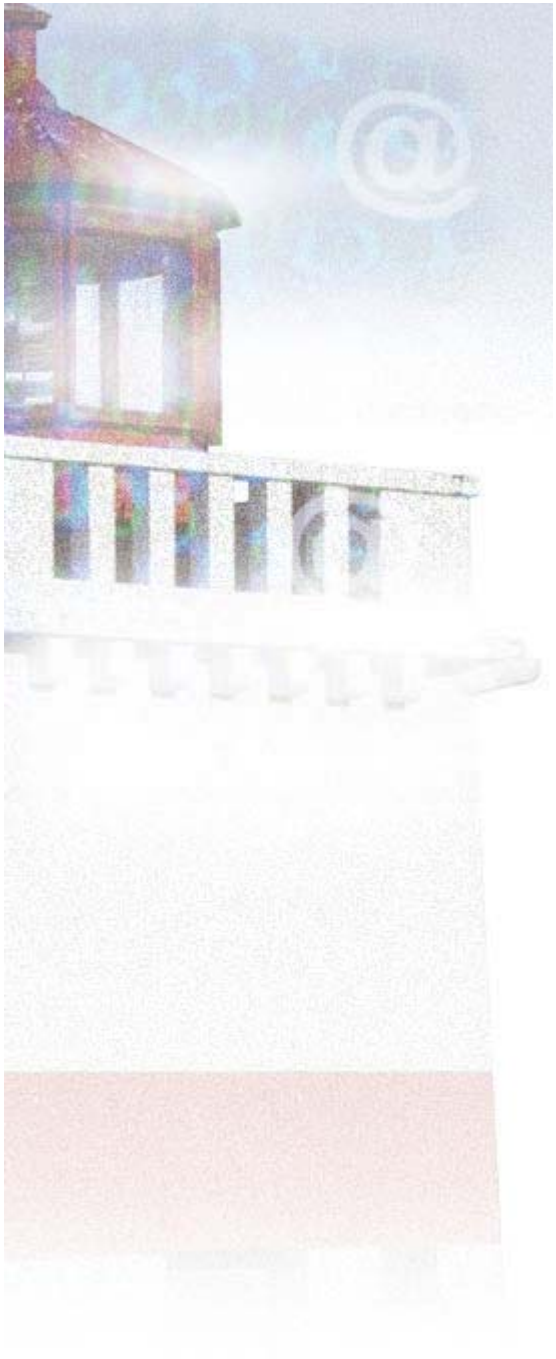
The 7400 chip, contains four NANDs

Tautology and contradiction

- *A tautology* is a proposition that is always true.
- *A contradiction* is a proposition that is always false.

p	$\neg p$	$p \vee \neg p$	$p \wedge \neg p$
0	1	1	0
1	0	1	0

Adapted from C. Heeren



Laws for propositional forms

- *Law of the excluded middle*

$$p \vee \neg p \equiv 1$$

- *Law of contradiction*

$$p \wedge \neg p \equiv 0$$

- *Double negation*

$$\neg(\neg p) \equiv p$$

Adapted from C. Heeren

Rules of replacement

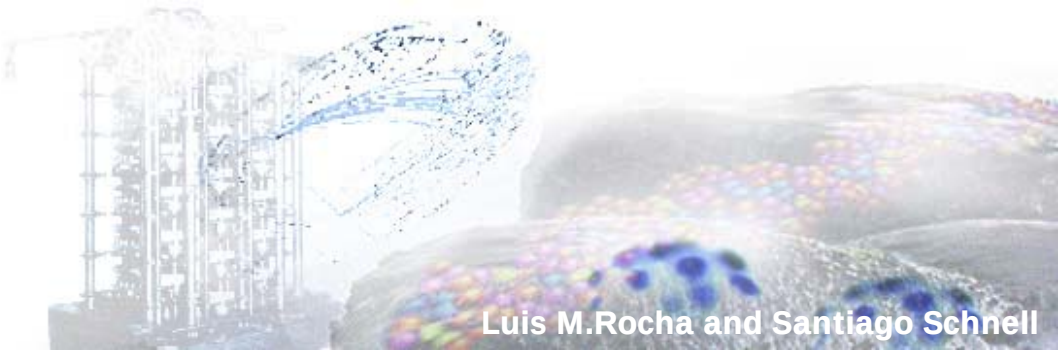
- *De Morgan's Law I*

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

- *De Morgan's Law II*

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

Adapted from C. Heeren



Proofs for De Morgan's Law I

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

p	q	$\neg p$	$\neg q$	$\neg p \wedge \neg q$	$p \vee q$	$\neg(p \vee q)$
0	0	1	1	1	0	1
0	1	1	0	0	1	0
1	0	0	1	0	1	0
1	1	0	0	0	1	0

Adapted from C. Heeren

Next Class!

- Topics
  - Overview for Midterm Exam
- Readings for Next week
  - @ *infoport*
  - From course package
    - Klir, J.G., U. St. Clair, and B.Yuan [1997]. Fuzzy Set Theory: foundations and Applications. Prentice Hall
      - Chapter 2: Classical Logic (pp. 87-98)
      - Chapter 3: Classical Set Theory (pp. 99-107)
- Lab 7
  - Intro to Statistical Analysis using Excel
  - NO LAB THIS WEEK!!!!