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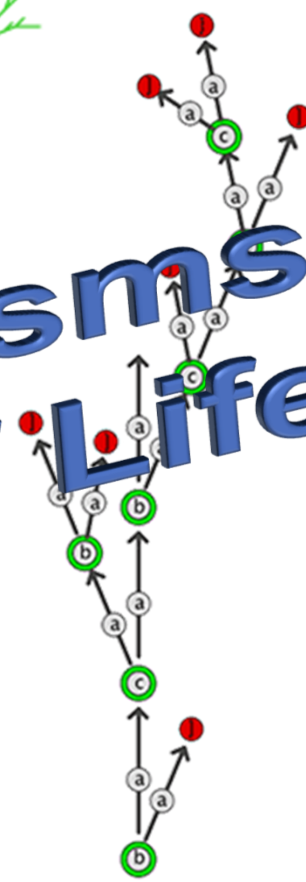
Modeling the logical mechanisms and organization of Life



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key events coming up

- Labs: 35% (ISE-483)
  - Complete 5 (best 4 graded) assignments based on algorithms presented in class
    - Lab 0 : February 3rd
      - *Introduction to Python* (No Assignment)
        - Delivered by Srikanth Iyer
          - see solved exercises!
    - Lab 1 : February 10th
      - *Measuring Information* (Assignment 1)
        - Delivered by Shayan Esfarayeni
        - Due February 17th.
    - Lab 2 : February 24th
      - L-Systems (Assignment 2)
        - Delivered by Rik Pardun
        - Due: March 3rd
- SSIE – 583 -Presentation and Discussion: 25%
  - Present and lead the discussion of an article related to the class materials
    - Enginet students post/send video or join by Zoom
  - Papers need to be assigned!

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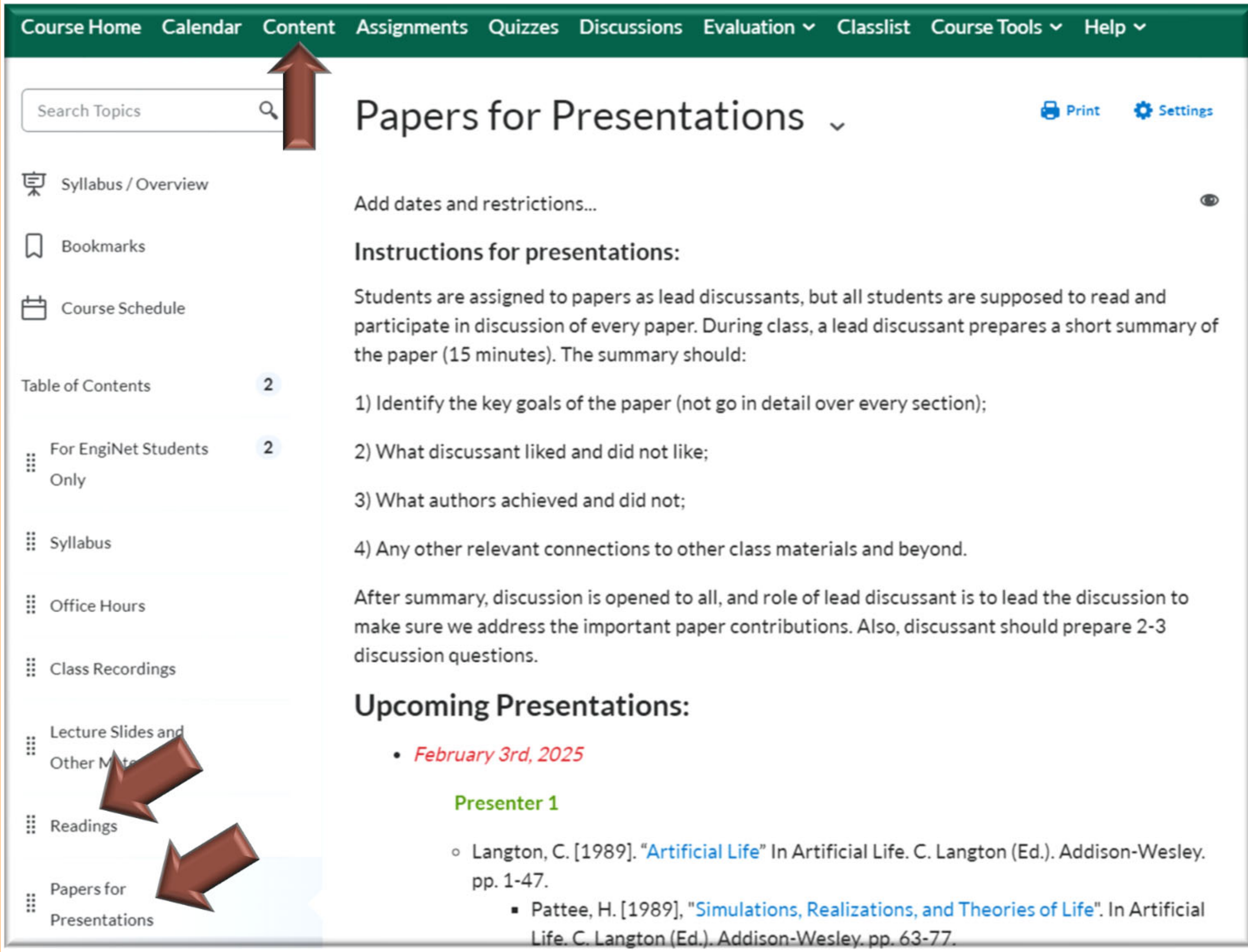
SSIE-583 - possible presentations

Some classics

- Adami, C. [2006]. “Digital Genetics: Unraveling the Genetic Basis of Evolution”. *Nature Reviews Genetics.* **7**:109-118.
- Conrad, M. [1990]. "The geometry of evolution.“ *Biosystems* **24**: 61-81
- Crutchfield, J.P. and M. Mitchell [1995]."The evolution of emergent computation." *Proc. National Academy of Sciences*, USA, **92**: 10742-10746.
- Hinton, G.E. and S.J. Nowlan [1987]."How learning can guide evolution." *Complex Systems*. **1**, pp.495-502.
- Kauffman, S.A. [1969]. "Metabolic stability and epigenesis in randomly constructed genetic nets". *Journal of Theoretical Biology* **22**(3):437-467.
- Langton, C. [1989]. “Artificial Life” In *Artificial Life*. C. Langton (Ed.). Addison-Wesley. pp. 1-47.
  - Pattee, H. [1989], "Simulations, Realizations, and Theories of Life". In *Artificial Life*. C. Langton (Ed.). Addison-Wesley. pp. 63-77.
- Lindgren, K. [1991]."Evolutionary Phenomena in Simple Dynamics." In: *Artificial Life II*. Langton et al (Eds). Addison-wesley, pp. 295-312.
- Ray, T. S. 1992. "Evolution, ecology and optimization of digital organisms". *Santa Fe Institute* working paper 92-08-042.
- Pattee, Howard H. [1969] "How does a molecule become a message?." *Communication in development* **3**: 1-16.
- Schmidt, M. and H. Lipson [2009]. “Distilling Free-Form Natural Laws from Experimental Data. *Science*, **324**: 81-85.
- Sims,K. [1994]. “Evolving Virtual Creatures”. *Proceedings of the 21st annual conference on Computer graphics and interactive techniques*, pp. 15 – 22.
  - H. Lipson and J. B. Pollack (2000), "Automatic design and Manufacture of Robotic Lifeforms", *Nature* **406**: 974-978.
  - Lipson H. (2005) "Evolutionary Design and Evolutionary Robotics", *Biomimetics*, CRC Press (Bar Cohen, Ed.) pp. 129-155
- Varela, Francisco J.; Maturana, Humberto R.; & Uribe, R. [1974]. “Autopoiesis: the organization of living systems, its characterization and a model”. *Biosystems.* **5** 187–196.

SSIE-583 - possible presentations

Some classics



Evolution". *Nature Reviews Genetics.*

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omputation." *Proc. National Academy of*

on." *Complex Systems*. **1**, pp.495-502.

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isms". *Santa Fe Institute* working paper

?." *Communication in development* **3**:

from Experimental Data. *Science*, **324**:

annual conference on Computer

Robotic Lifeforms", *Nature* **406**: 974-978.

netics, CRC Press (Bar Cohen, Ed.) pp. 129-

topoiesis: the organization of living

until now

- Class Book
  - Floreano, D. and C. Mattiussi [2008]. *Bio-Inspired Artificial Intelligence: Theories, Methods, and Technologies*. MIT Press. **Preface**.
    - Nunes de Castro, Leandro [2006]. *Fundamentals of Natural Computing: Basic Concepts, Algorithms, and Applications*. Chapman & Hall. **Chapter 1,** pp. 1-23.
- Lecture notes
  - Chapter 1: “What is Life?”
  - Chapter 2: The logical Mechanisms of Life
    - posted online @ http://informatics.indiana.edu/rocha/i-bic

- Papers for Presentations
  - Logical mechanisms of life (optional for SSIE 483)
    - Langton, C. [1989]. “Artificial Life” In *Artificial Life*. C. Langton (Ed.). Addison-Wesley. pp. 1-47.
      - Pattee, H. [1989], "Simulations, Realizations, and Theories of Life". In Artificial Life. C. Langton (Ed.). pp. 63-77
- Other Readings
  - Optional
    - Gleick, J. [2011]. *The Information: A History, a Theory, a Flood*. Random House. **Chapter 8**.
    - Cobb, Matthew. [2013]. "1953: When Genes Became 'Information'." *Cell* **153** (3): 503-506.
    - Aleksander, I. [2002]. “Understanding Information Bit by Bit”. In: *It must be beautiful : great equations of modern science*. G. Farmelo (Ed.), Grant
    - James, R., and Crutchfield, J. (2017). Multivariate Dependence beyond Shannon Information. Entropy, 19(10), 531.
    - Prokopenko, Mikhail, Fabio Boschetti, and Alex J. Ryan. "An information-theoretic primer on complexity, self-organization, and emergence." Complexity 15.1 (2009): 11-28.

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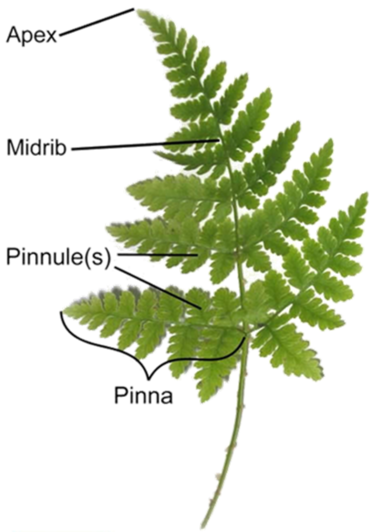
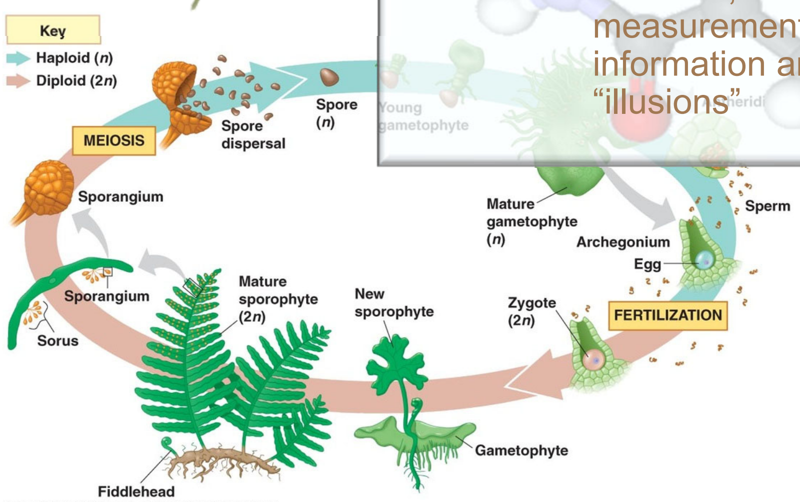
alternative concepts of mechanism

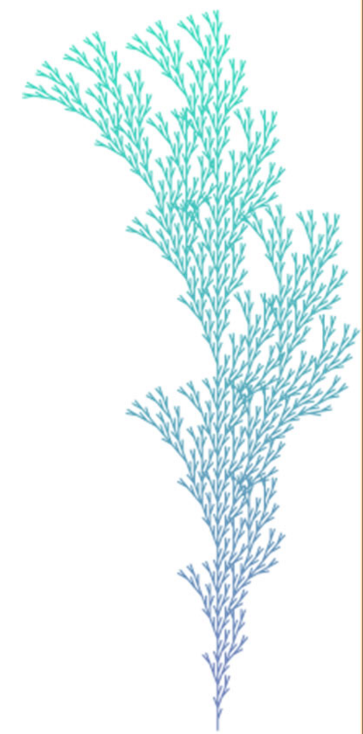


- Analytical
  - Reduction to (non-living) components
    - Reductionism
  - ***Material*** mechanism
  - Life is complicated chemistry
  - Tied to specific materiality
  - Does not allow emergence (in a radical "strawman" view)
    - Function, control, measurement, categorization, information are unnecessary "illusions"

- Synthetic
  - Construction from components
    - Holist
  - ***Logical*** mechanism
  - Life is **Organization**
    - **Networks of components**
  - Universal or implementation independent
  - Emergence
    - "bottom-up" approach



Luis M. Rocha. SSIE 483X/583X - Lecture Notes - Chapter 1:What is Life? and Chapter 2: The logical mechanisms of life.



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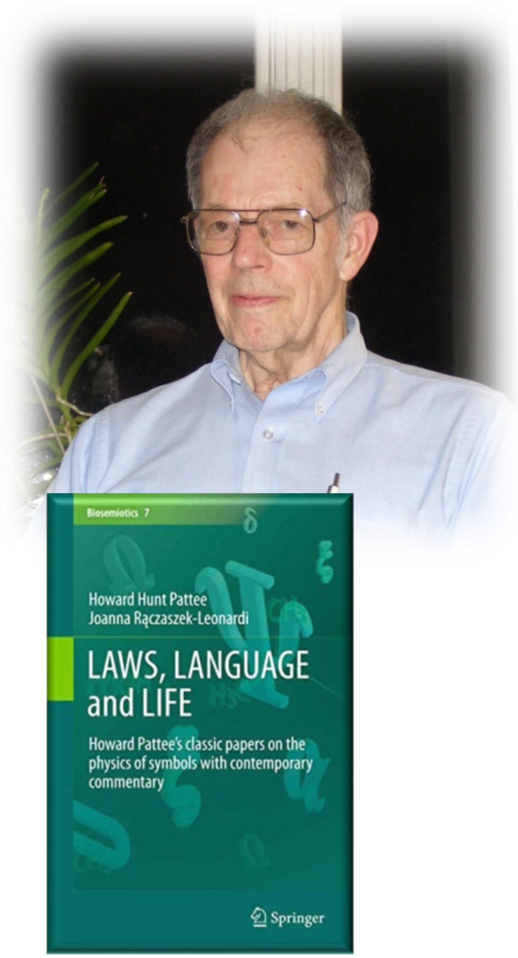
criteria for deciding good simulations or realizations?



- Alife must be compared to something
  - What is the formal/logical threshold of complexity?
    - **Hard Alife** must provide a set of rules to distinguish Alife from artificial matter
    - **Weak Alife** needs to be able to test design principles of life with simulations
      - **Bio-inspired computing** needs only to produce good results in engineering problems
  - Comparison to “life-like” behavior is too subjective
- theories of life
  - Alife methodology requires existing theories of life to be compared against
    - constrained by (rather than freed from) our theories or “fiction”
  - contributes to the meta-methodology of Biology
    - test and improve beyond material constraints, such as the incomplete fossil record or measurement of cellular activity

Pattee, H. [1989], "Simulations, Realizations, and Theories of Life". In *Artificial Life*. C. Langton (Ed.) Addison-Wesley. pp. 63-77.

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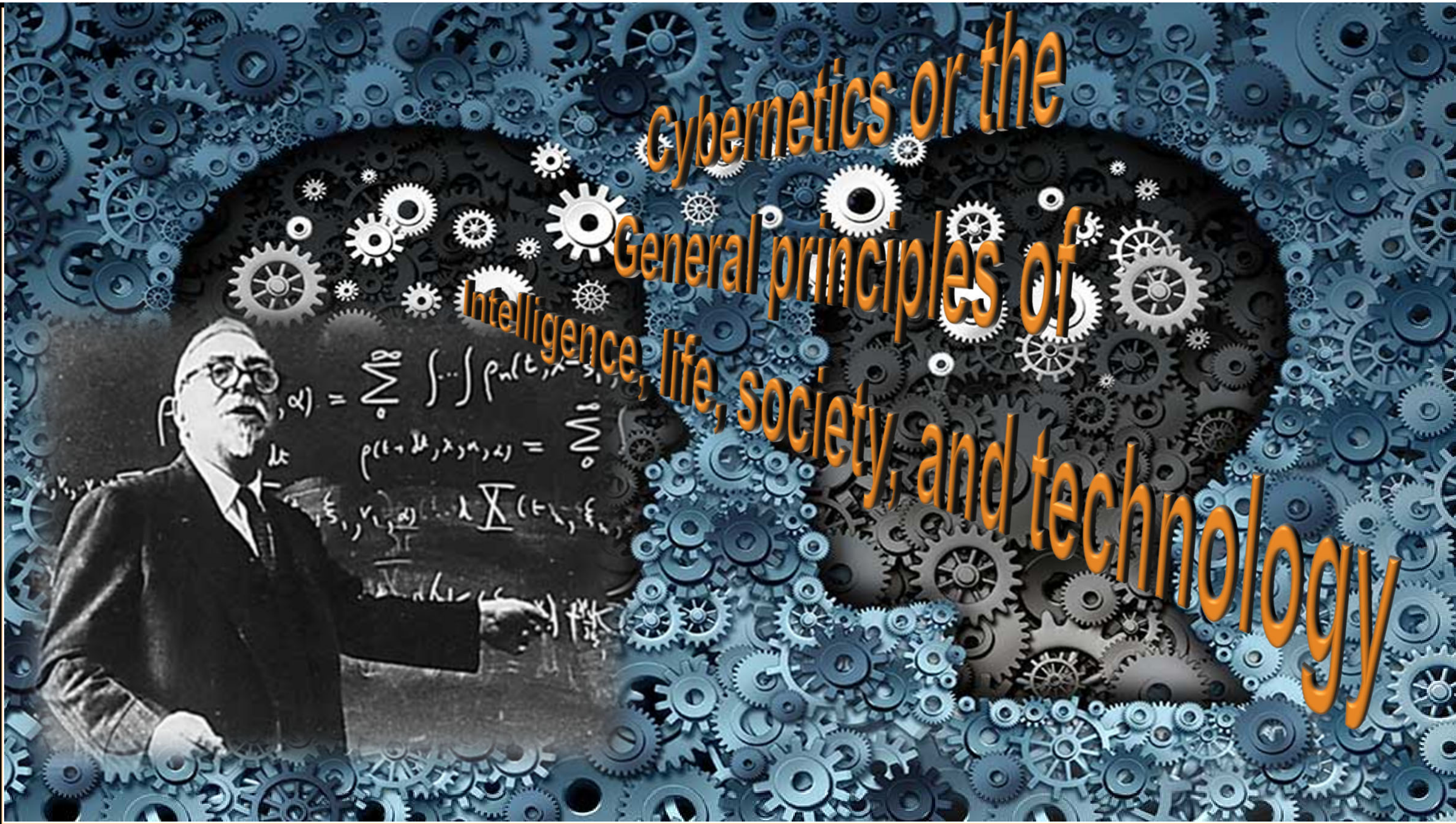
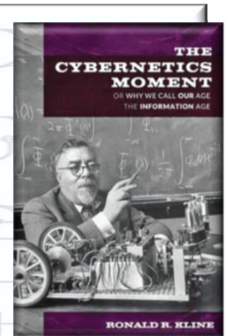
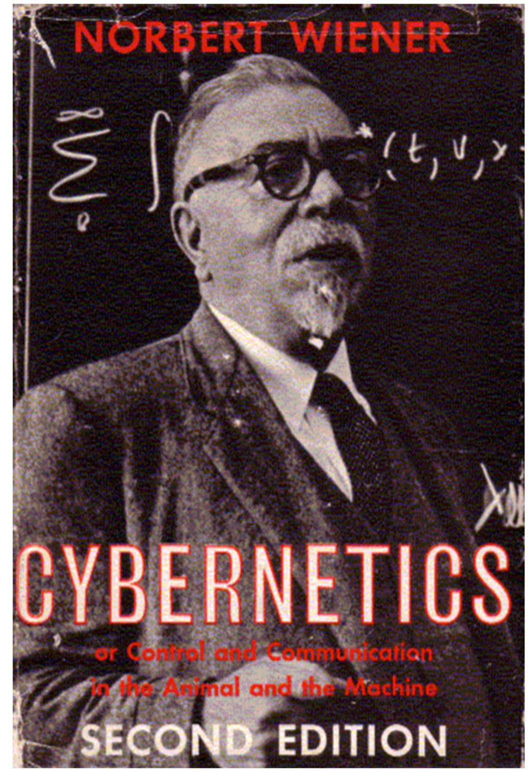

Cybernetics or the
General principles of
Intelligence, life, society, and technology

post-war science

- Synthetic approach
  - Engineering-inspired
  - Supremacy of mechanism
- Postwar culture of problem solving
  - Interdisciplinary teams
  - Cross-disciplinary methodology
- All can be axiomatized and computed
  - Mculloch&Pitts' work was major influence
    - "A logical calculus of the ideas immanent in nervous activity". *Bulletin of Mathematical Biophysics* **5**:115-133 (1943).
    - A **Turing machine** (any function) could be implemented with a **network** of **simple binary switches** (if circularity/feedback is present)





Warren S. McCulloch

Margaret Mead

Claude Shannon

Heinz Von Foerster

Walter Pitts

Macy Conferences: 1946-**53**

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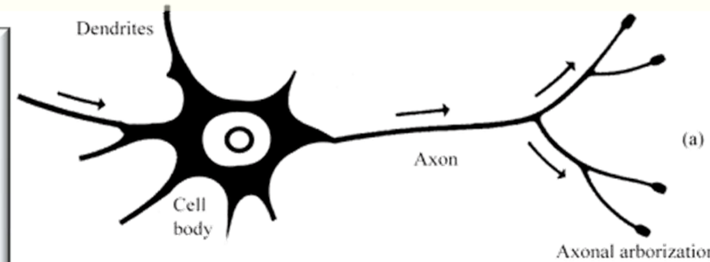
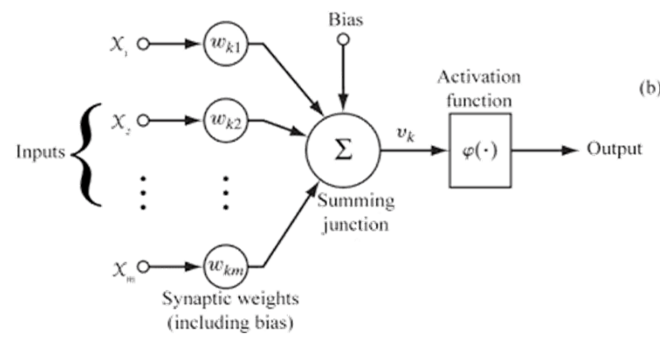
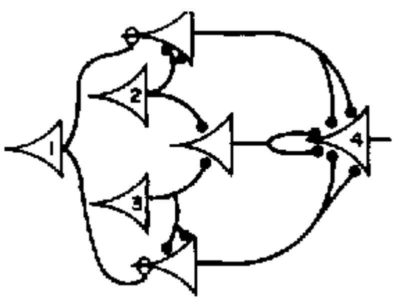
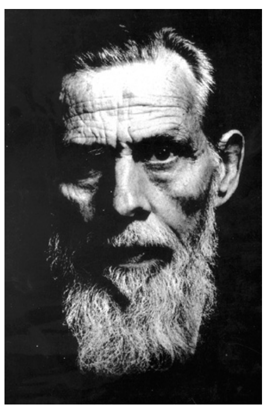
Memory can be maintained in circular networks of binary switches

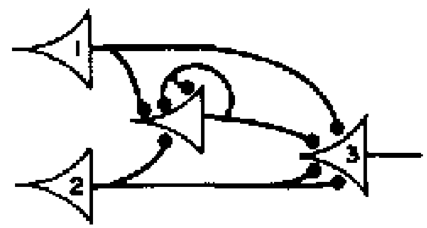
- McCulloch, W. and W. Pitts [1943], "A Logical Calculus of Ideas Immanent in Nervous Activity". *Bulletin of Mathematical Biophysics* **5**:115-133.
  - A Turing machine program could be implemented in a finite network of binary neuron/switches
    - Neurons as basic computing unit of the brain
    - Circularity is essential for memory (closed loops to sustain memory)
  - Brain (mental?) function as computing
- Others at Macy Meeting emphasized other aspects of brain activity
  - Chemical concentrations and field effects (not digital)
    - Ralph Gerard and Fredrik Bremmer

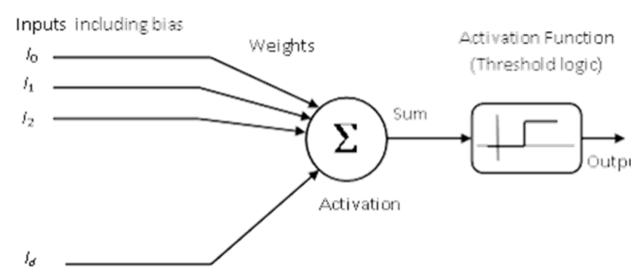












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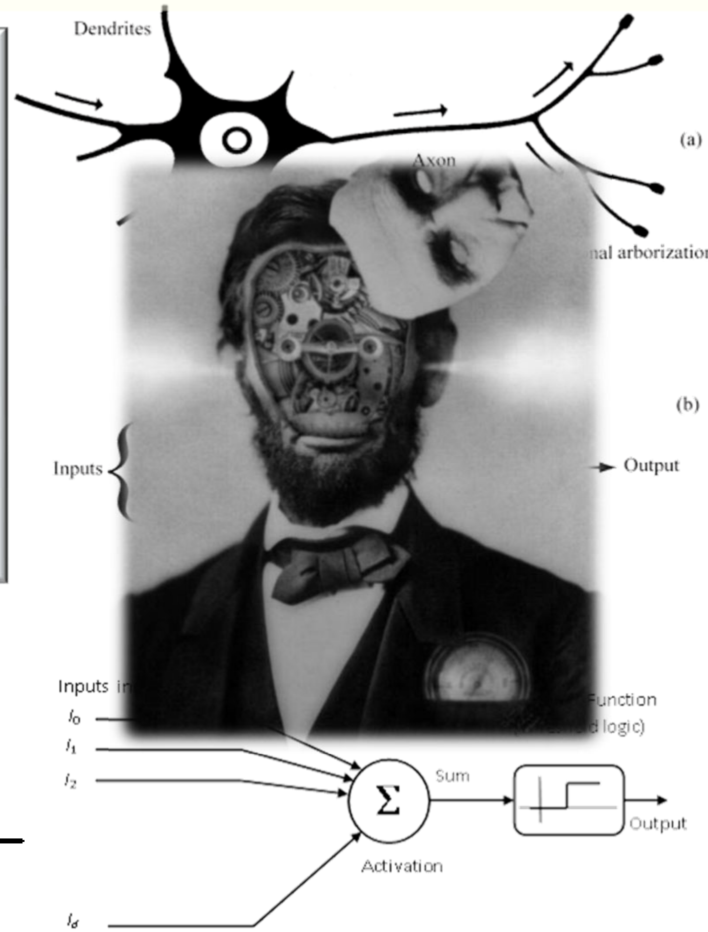

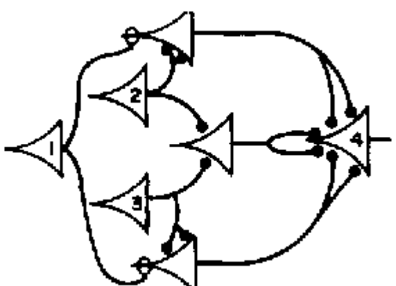

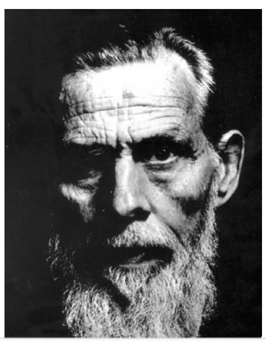

dominicm73.blogspot.com/

McCulloch, W. and W. Pitts [1943], "A Logical Calculus of Ideas Immanent in Nervous Activity". *Bulletin of Mathematical Biophysics* **5**:115-133.

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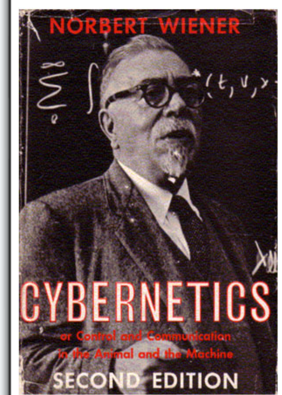
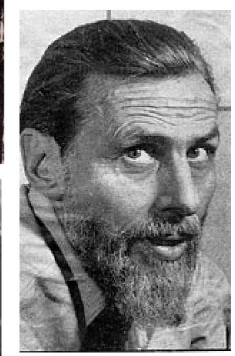
universal computers and general-purpose mechanisms and organization

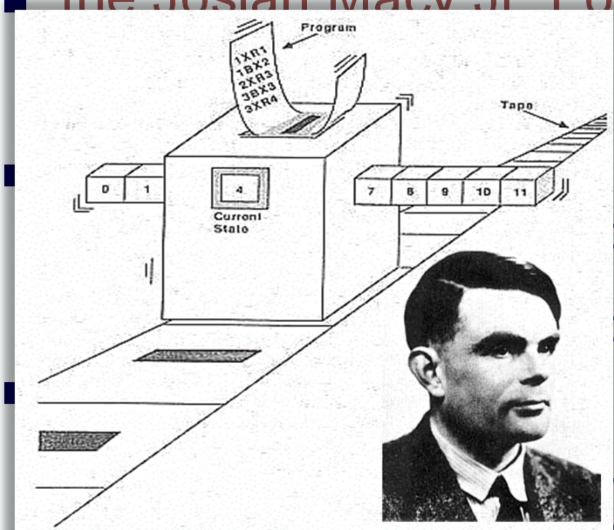
- the Josiah Macy Jr. Foundation Meetings
  - post-war science
    - 1946-1953
- Interdisciplinary
  - Since a large class of ordinary phenomena exhibit circular causality, and mathematics is accessible, let's look at them with a war-time team culture
- Participants
  - **John Von Neumann**, Leonard Savage, **Norbert Wiener**, **Arturo Rosenblueth**, Walter Pitts, **Margaret Mead**, Heinz von Foerster, **Warren McCulloch**, **Gregory Bateson**, Claude Shannon, Ross Ashby, etc.
- Key concepts
  - **universal computation** (Turing, Von Neumann), **information** (Shannon, Wiener), **networks** (mcCulloch), homeostasis, **feedback**, complexity, self-organization
  - mind, society, life as general mechanisms and organization





universal computers and general-purpose mechanisms and organization

- the Josiah Macy Jr. Foundation Meetings



ordinary phenomena e
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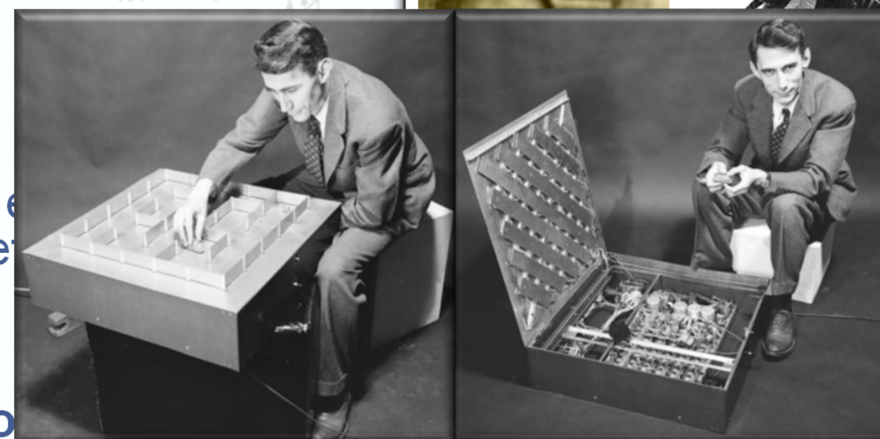
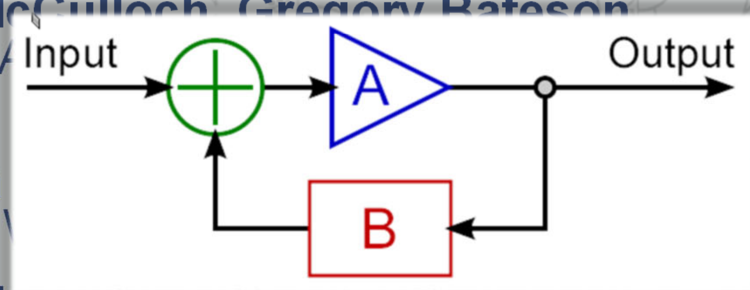
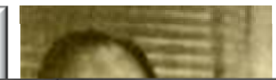
culture

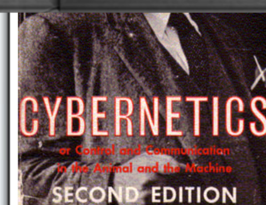
Leonard Savage, No
Walter Pitts, **Margaret Mead**, Heinz
von Foerster, **Warren McCulloch**, **Gregory Bateson**,
Claude Shannon, Ross A

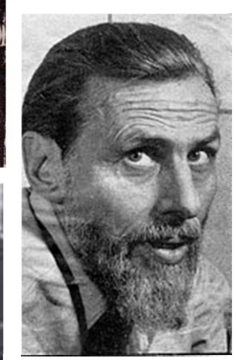




- Key concepts
  - **universal computation**
    information (Shannon, W
    homeostasis, **feedback**,
  - mind, society, life as general mechanisms and organization





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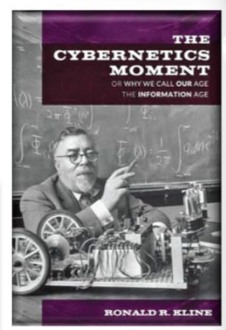
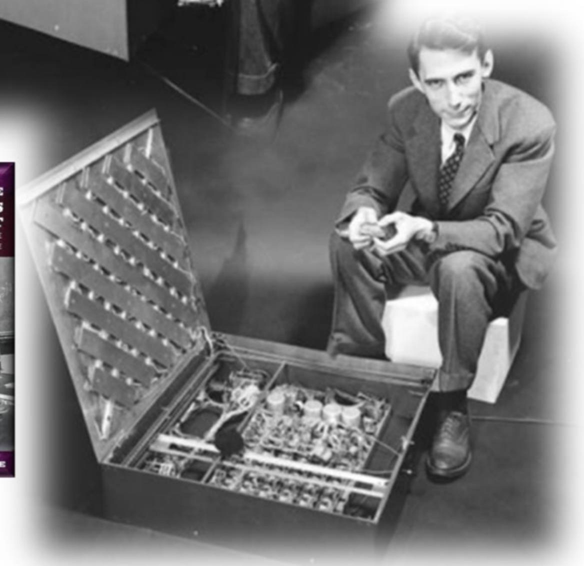
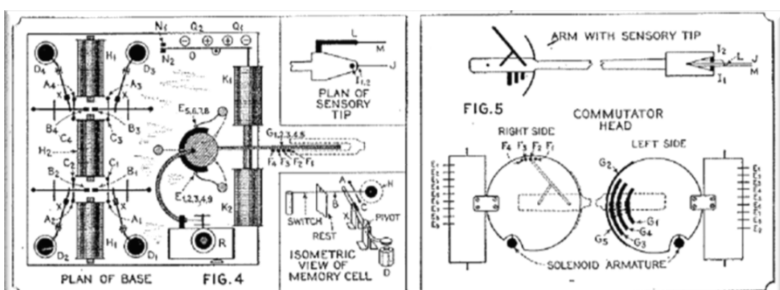
controlling information to achieve life-like behavior

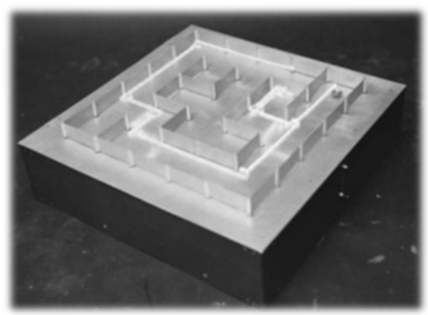
- trial and error algorithm
  - information as reduction of uncertainty in the presence of alternatives (combinatorics)
- lifelike behavior
  - trial and error to **learn** path from many alternatives
  - adapts to new situations
- how is learning achieved?
  - Correct choices, **information** gained from reduced uncertainty, must be **stored in memory**
- **memory of information** as a design principle of intelligence in uncertain environments
  - 75 bit memory
  - stored in (telephone) switching relays
    - Brain as (switching) machine







Details of the various parts of the "memory cells" and actuating mechanisms



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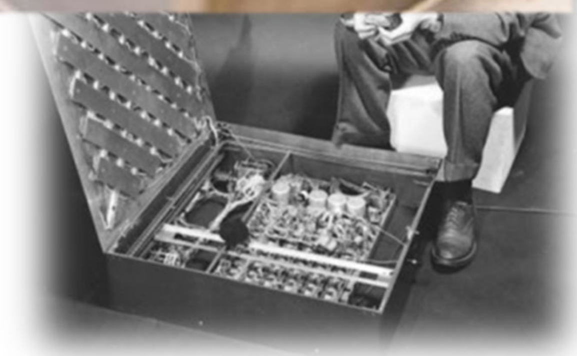
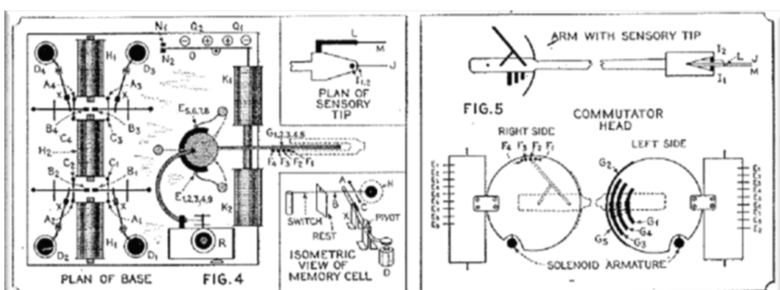
controlling information to achieve life-like behavior





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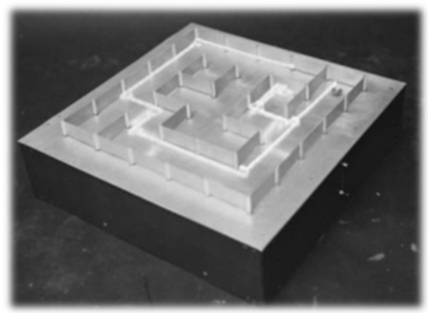

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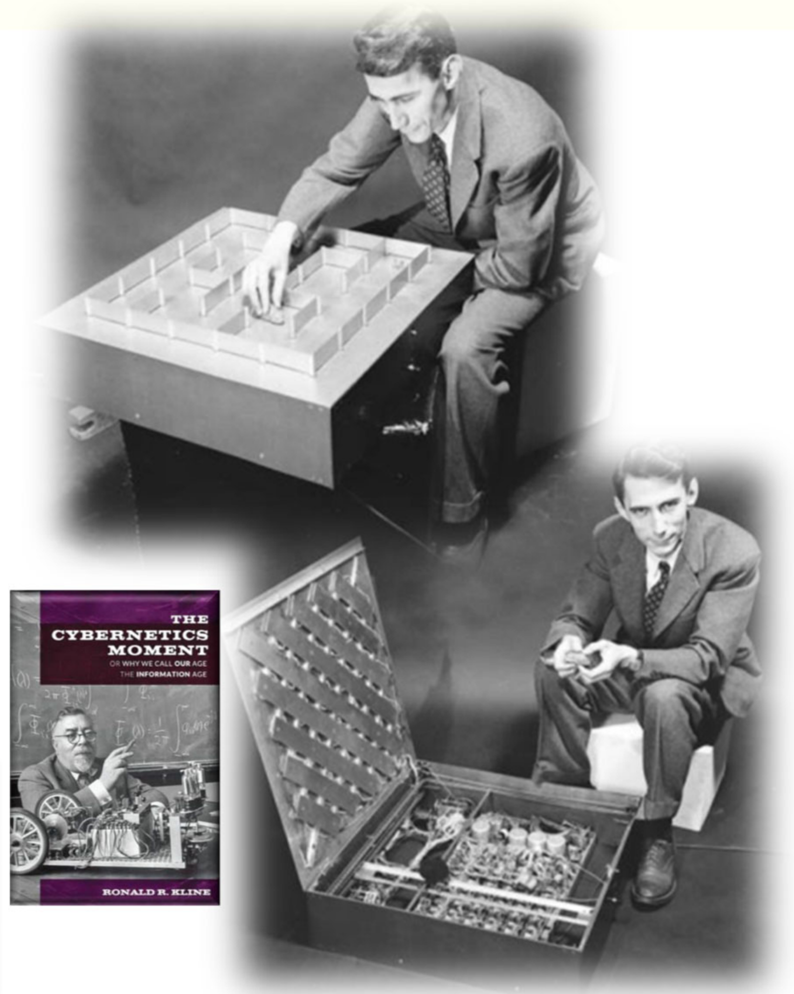
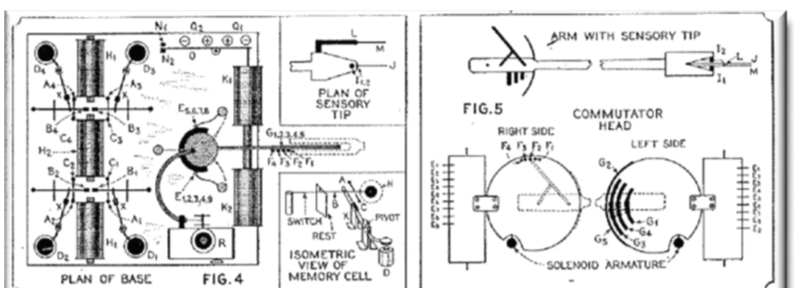
Shannon’s mouse

controlling information to achieve life-like behavior





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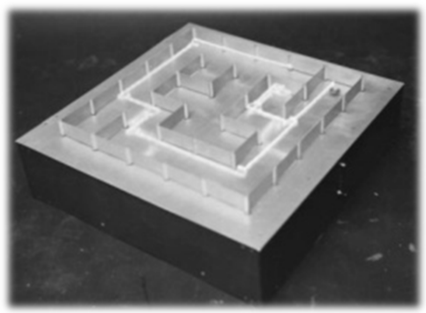

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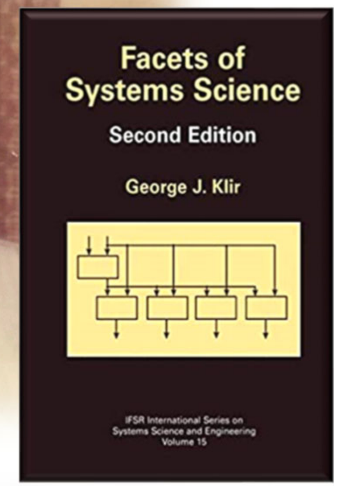
a science of organization across disciplines

- Systemhood properties of nature
  - Robert Rosen
    - Systems depends on a specific adjective: **thinghood**
    - **Systemhood**: properties of arrangements of items, independent of the items
      - Similar to “setness” or cardinality
  - George Klir
    - **Organization** can be studied with the mathematics of **relations**
    - $S = (T, R)$
      - S: a System, T: a set of things(thinghood), R: a (or set of) relation(s) (Systemhood)
      - Same relation can be applied to different sets of objects
      - Systems science deals with **organizational properties** of systems independently of the items
    - Examples
      - Collections of books or music files are sets of things
      - But organization of such sets are systems (alphabetically, chronologically, typologically, etc.)



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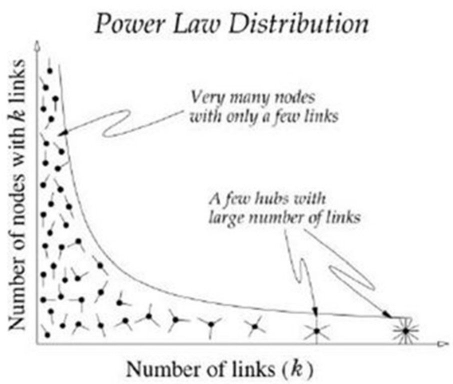
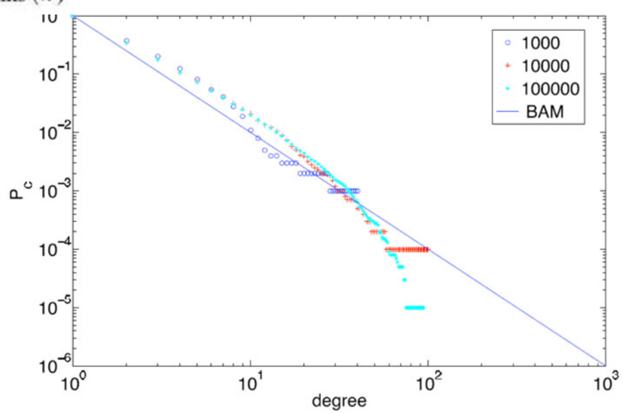
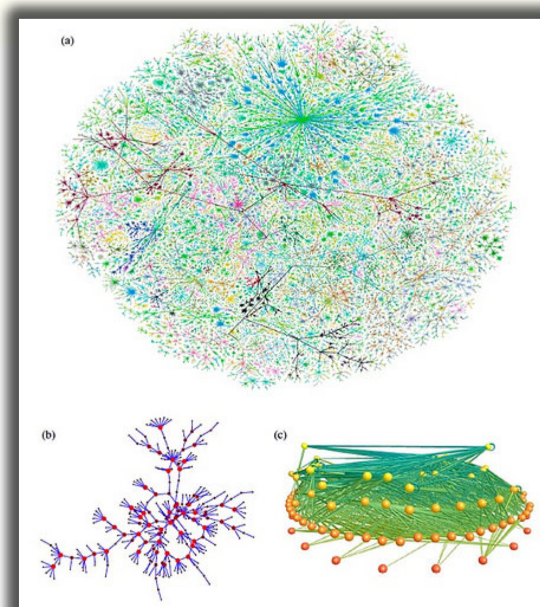
example of general principle of organization

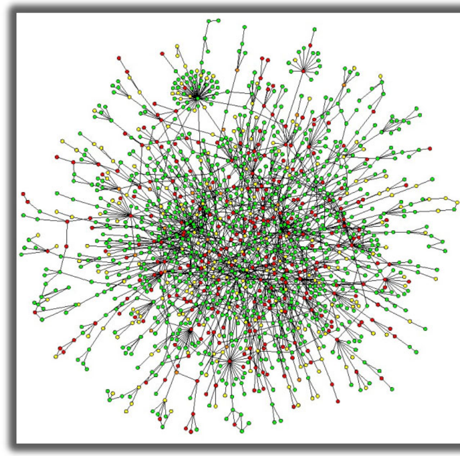
Barabasi-Albert Model: leads to power-law node degree distributions in networks

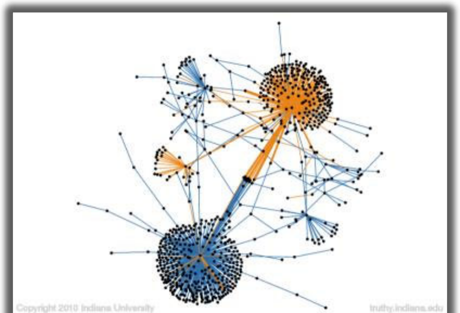
Amaral et al: Most real networks have a cut-off distribution for high degree nodes which can be computationally modeled with vertex aging.

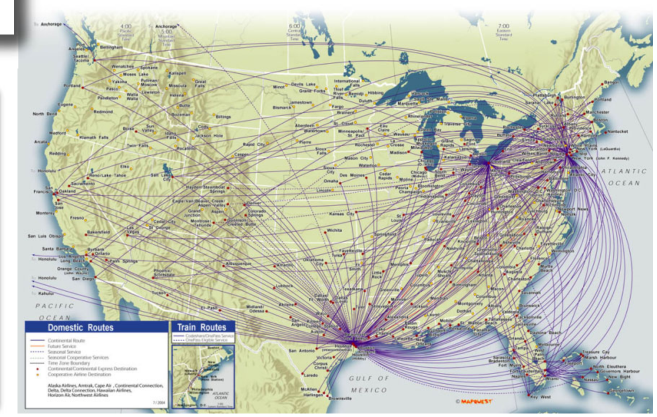












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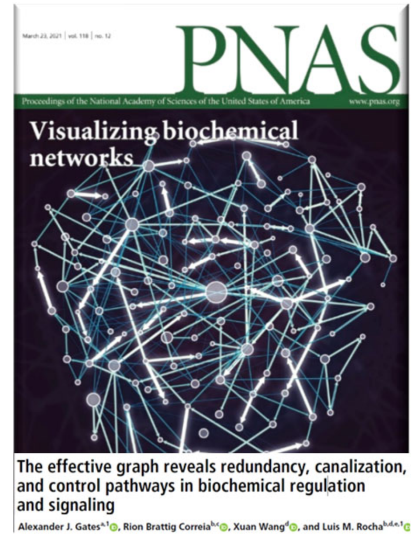
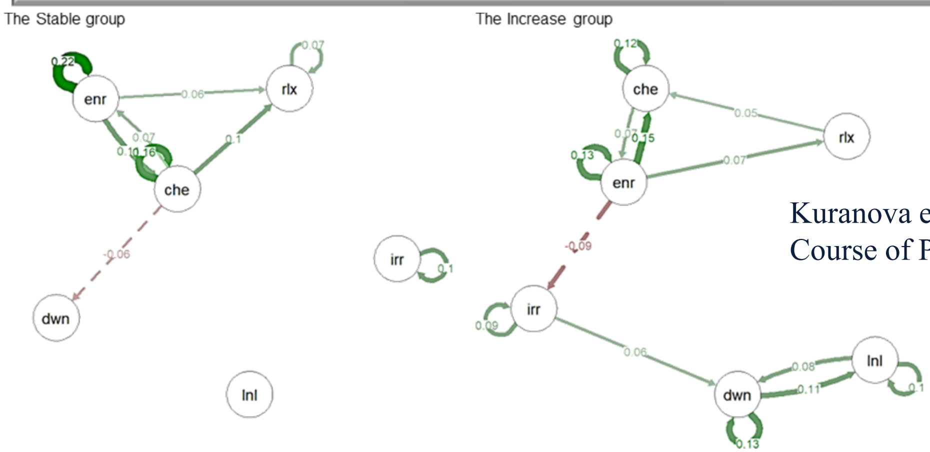
“life-as-it-could-be” is the *systemhood* of life

- A system possesses ***systemhood*** and ***thinghood*** properties
  - *Thinghood* refers to the specific material that makes up the system
  - *Systemhood* are the abstracted, organizational properties
    - E.g. a clock can be made of different things, but there are implementation-independent properties of “clockness”
    - E.g. Evolutionary systems are organized according to a genotype/phenotype map, but such maps do not need to be made of DNA and Protein
      - Langton’s extended Gtype/Ptype
  - Systems science deals with the implementation-independent aspects of systems
    - Allows the conceptualization of unobserved organizations, e.g. “life-as-it-could-be”
      - E.g. networks of logical units to represent biochemical (or psychopathology) regulation and dynamics
    - But systems science is supposed to be validated empirically on thinghood
      - Otherwise it is mathematics or sophisticated thought experiments (computational philosophy)





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Kuranova et al. [2021] “Network Dynamics of Momentary Affect States and Future Course of Psychopathology in Adolescents.” *PLOS ONE* 16(3): e0247458.

Bringmann et al. [2022] "Psychopathological Networks: Theory, Methods and Practice." *Behaviour Research and Therapy* **149**: 104011.

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    - But systems science is supposed to be validated empirically on thinghood
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  - Criticisms
    - Circumscribed to theories of life (reality is stranger than fiction)
    - Alife rarely goes beyond showing artificial behavior that resembles real life
    - The role of materiality: embodiment in life as major feature of evolutionary system
      - evolutionary robotics, embodied cognition

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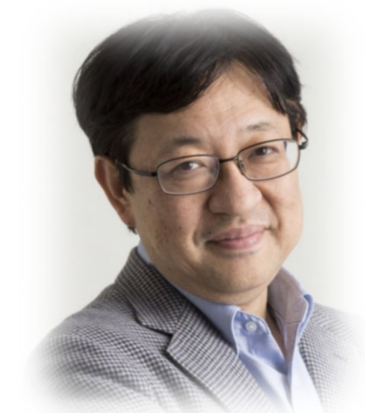
post-reductionism synthetic approaches

- Reductionism in Biology (analysis)
  - search and characterization of the *function* of building blocks (genes and molecules)
- Post-genome informatics
  - Minoro Kanehisa: biology is moving onto synthesis from structural and functional genomics
- Computational and systems biology
  - Non-reductionist modeling of life from analysis of large-scale biochemical information
    - Synthesis of biological knowledge from genomic **information**
      - The genome contains information about building blocks but it is naive to assume that it also contains the information on how the building blocks relate, develop, and evolve.
- Biomedical complexity pursued as systems modeling but tested in "life as we know it"
  - Towards an interdisciplinary understanding of basic ***principles*** of life via the search and characterization of networks of building blocks (genes and molecules)
    - Systems biology embraces the view that most interesting human organism traits such as immunity, development and even diseases such as cancer arise from the *operation of complex biological systems or networks.*
    - Multilevel regulation and signaling networks in health and disease
      - E.g. social determinants of health, epidemiology
    - Systems concepts such as control, modularity, networks, information and hierarchies
  - Grand (Modeling) Challenge
    - Given a complete genome sequence, reconstruct (synthesize) in a computer the functioning of a biological organism
    - Synthetic as artificial life, but grounded to "life-as-observed."



Kitano, Hiroaki. "Systems biology: a brief overview." *Science* 295.5560 (2002): 1662-1664
Villa, A. & S.T. Sonis. "System biology." In *Translational Systems Medicine and Oral Disease*, pp. 9-16. Academic Press, 2020.

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- Biomedical co… but …
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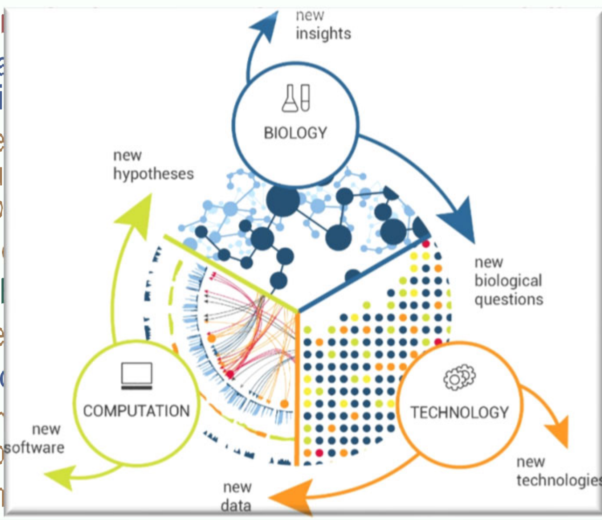

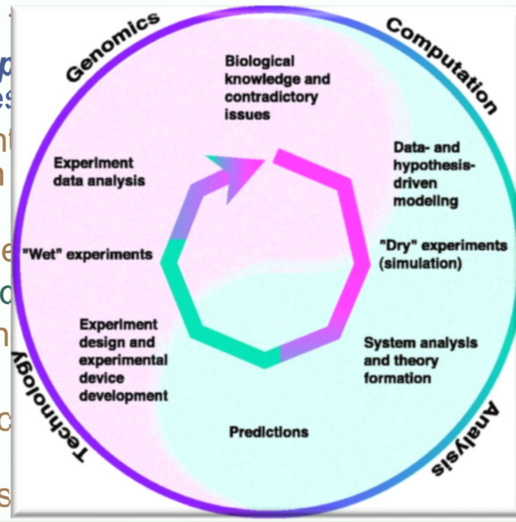

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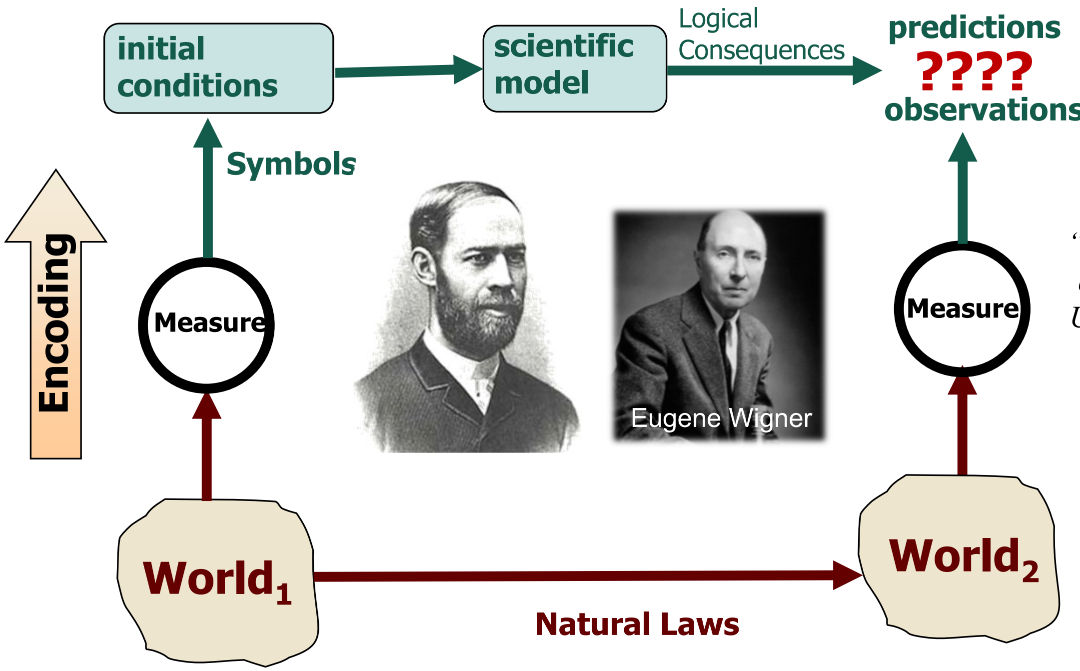
Hertzian scientific modeling paradigm



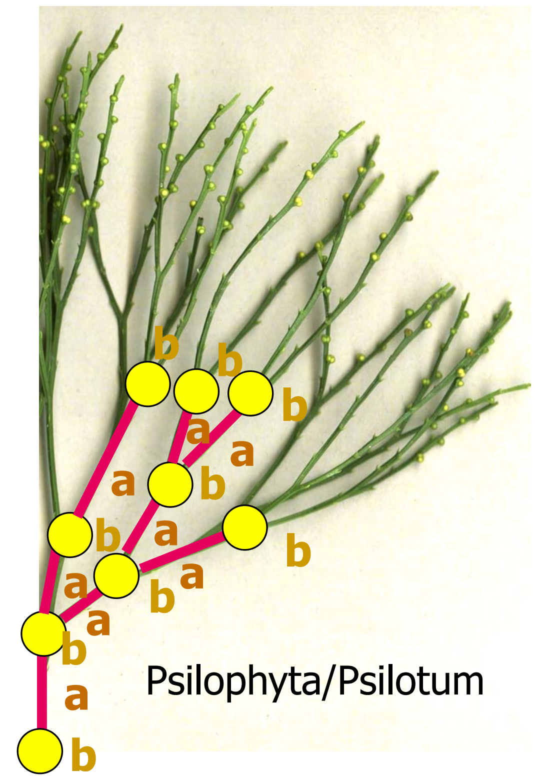
*"**Every empirical law has the disquieting quality that one does not know its limitations**." E. Wigner [1957] in "The Unreasonable Effectiveness of Mathematics in the Natural Sciences"*

"The most direct and in a sense the most important problem which our conscious knowledge of nature should enable us to solve is the ***anticipation of future events***, so that we may arrange our present affairs in accordance with such anticipation". (Hertz, 1894)

Building models

- What do you see?
  - Plants typically ***branch*** out
  - How can we ***model*** that?
    - Observe the distinct parts
      - Color them
      - Assign *symbols*
    - Build Model
      - Initial State: b
      - b -> a
      - ~~a -> b~~
      - a -> ba
    - Does not model all!

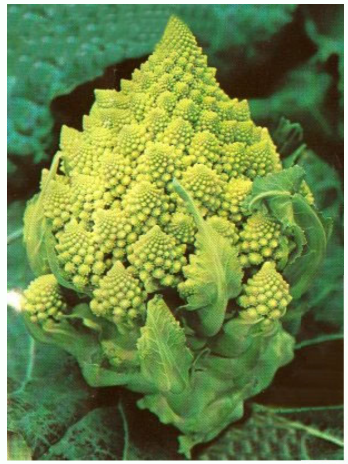

```
            b
            |
            a
           _|_
         a     b
       _|       \
      a  b        a
    _|   |        |_
    a  b  a       a  b
  _/   |  |_      |_   \
 a  b   a  a  b   a  b   a
```

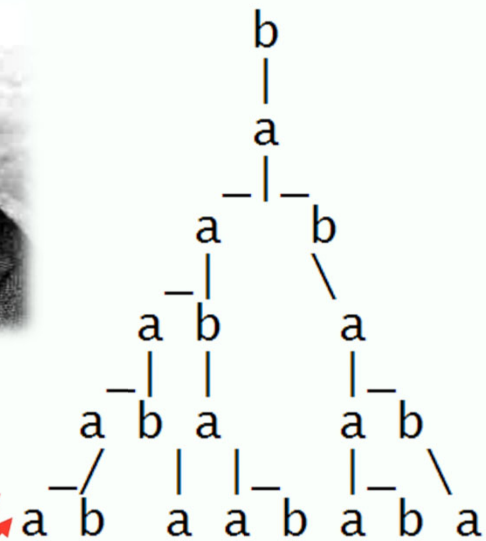


Fibonacci Numbers!

our first model of life

- ***Rewriting*** **production rules**
  - Initial State: b
  - b -> a
  - a -> ba
    - n=0 : b
    - n=1 : a
    - n=2 : ba
    - n=3 : aba
    - n=4 : baaba
    - n=5 : aababaab
    - n=6 : babaabaababaa
    - n=7 : abaababaabaababaababa
  - The length of the string is the Fibonacci Sequence
    - 1 1 2 3 5 8 13 21 34 55 89 ...
  - Fibonacci numbers in Nature
    - https://sciber.blogspot.com/2022/09/modeling-systems.html
    - Romanesco: https://www.wussu.com/fractals/romanesco.htm





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our first model of life

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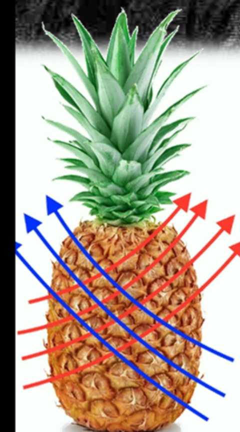
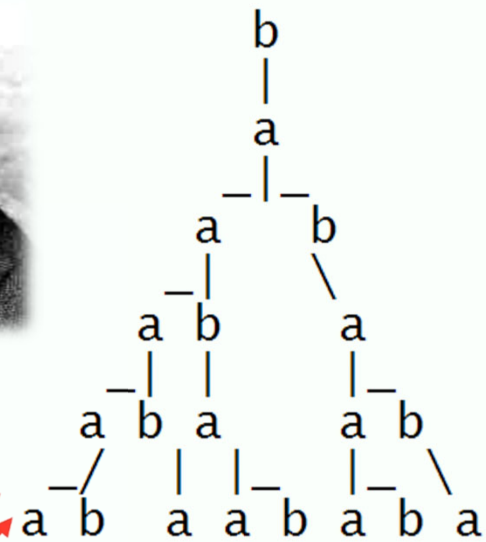
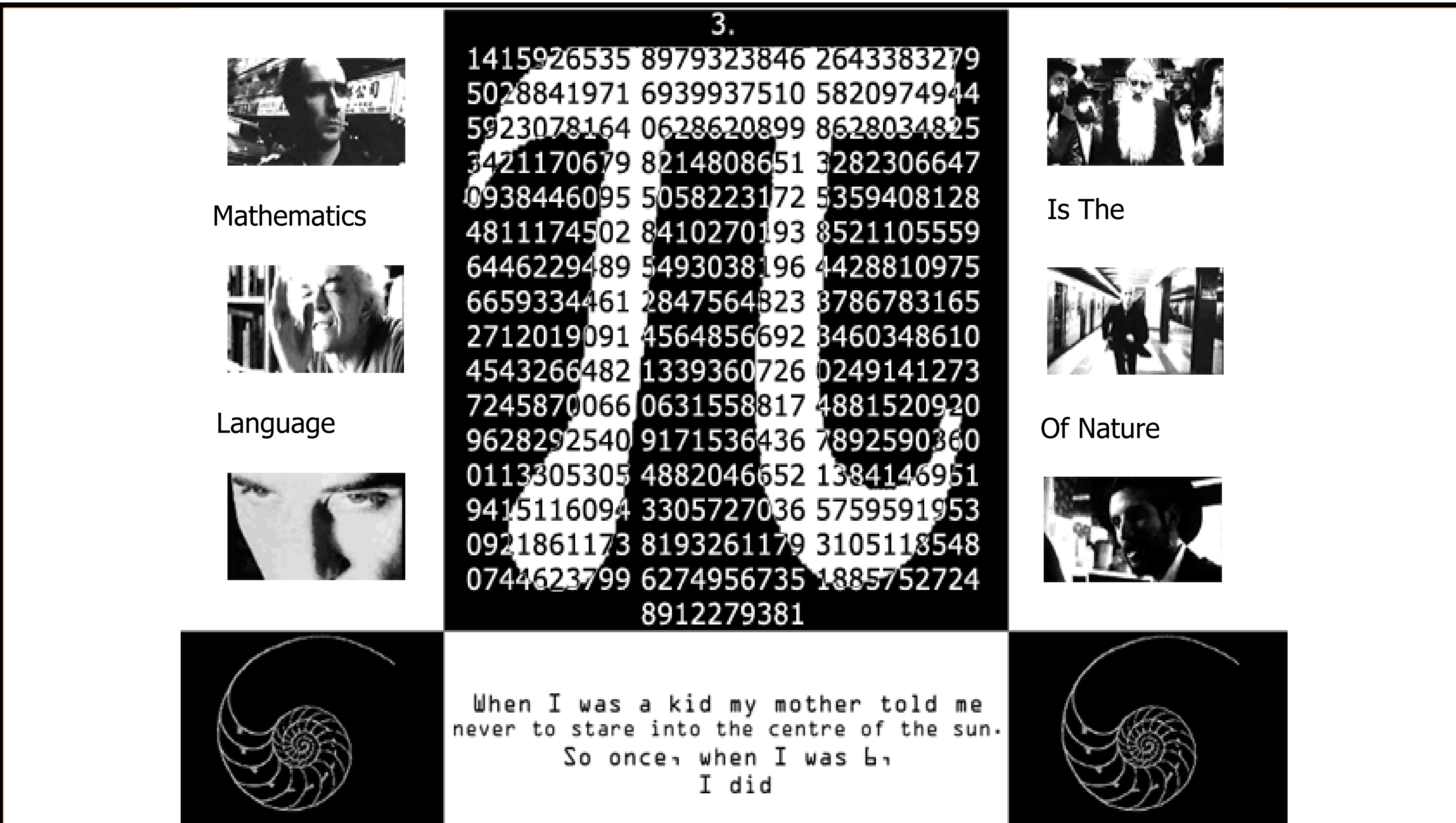
[redacted]

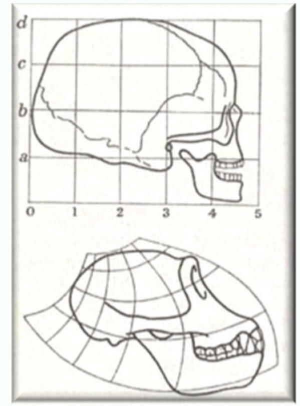





Mathematics
Language
3.
1415926535 8979323846 2643383279
5028841971 6939937510 5820974944
5923078164 0628620899 8628034825
3421170679 8214808651 3282306647
0938446095 5058223172 5359408128
4811174502 8410270193 8521105559
6446229489 5493038196 4428810975
6659334461 2847564823 3786783165
2712019091 4564856692 3460348610
4543266482 1339360726 0249141273
7245870066 0631558817 4881520920
9628292540 9171536436 7892590360
0113305305 4882046652 1384146951
9415116094 3305727036 5759591953
0921861173 8193261179 3105118548
0744623799 6274956735 1885752724
8912279381
Is The
Of Nature
When I was a kid my mother told me
never to stare into the centre of the sun.
So once, when I was 6,
I did

readings

- Class Book
  - Floreano, D. and C. Mattiussi [2008]. *Bio-Inspired Artificial Intelligence: Theories, Methods, and Technologies*. MIT Press. **Preface, Sections 4.1, 4.2**.
    - Nunes de Castro, Leandro [2006]. *Fundamentals of Natural Computing: Basic Concepts, Algorithms, and Applications*. Chapman & Hall. **Chapter 1,** pp. 1-23. Chapter 7, sections **7.1, 7.2** and **7.4** – Fractals and L-Systems, **Appendix B.3.1** – Production Grammars
- Lecture notes
  - Chapter 1: What is Life?
  - Chapter 2: The logical Mechanisms of Life
  - Chapter 3: Formalizing and Modeling the World
    - posted online @ casci.binghamton.edu/academics/i-bic



- Papers and other materials
  - Optional
    - Wigner, E.P. [1960], "The unreasonable effectiveness of mathematics in the natural sciences". *Comm. Pure Appl. Math.*, **13**: 1-14.
    - Flake's [1998], *The Computational Beauty of Life*. MIT Press.
      - Chapter **1** – Introduction
      - Chapters **5, 6** (7-9) – Self-similarity, fractals, L-Systems