lecture 5


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Recursion,
self-similarity and
L-System models

key events coming up

- Labs: 35% (ISE-483)
  - Complete 5 (best 4 graded) assignments based on algorithms presented in class
    - Lab 1 : February 10th
      - *Measuring Information* (Assignment 1)
        - Delivered by Shayan Esfarayeni
        - **Due February 17th.**
    - Lab 2 : February 24th
      - L-Systems (Assignment 2)
        - Delivered by Rik Pardun
        - Due: March 3rd
- SSIE – 583 -Presentation and Discussion: 25%
  - Present and lead the discussion of an article related to the class materials
    - Enginet students post/send video or join by Zoom
  - Papers need to be assigned!

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- Projects
  - **Due by May 7th** in Brightspace, “Final Project 483/583” assignment
    - ALIFE 2025
      - Not necessarily to submit to actual conference due date
        - May 4 full paper, July 4, abstract
      - https://2025.alife.org/
      - Max 8 pages, author guidelines:
      - https://2025.alife.org/calls#paper-call
      - MS Word and Latex/Overleaf templates
    - Preliminary ideas **by March 7**
      - Submit to “Project Idea” assignment in Brightspace.
  - Individual or group
    - With very definite tasks assigned per member of group

ALIFE 2025

Tackle a real problem using bio-inspired algorithms, such as those used in the labs.

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Reusing and expanding labs is highly encouraged.

schedule

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- March 3rd, 2025
  - Emad Abed
    - Scheffer, Marten, et al. "Early-warning signals for critical transitions." *Nature* **461**.7260 (2009): 53.
      - Optional: Leemput, Ingrid A van de, Marieke Wichers, Angélique O J Cramer, Denny Borsboom, Francis Tuerlinckx, Peter Kuppens, Egbert H van Nes, et al. "Critical Slowing down as Early Warning for the Onset and Termination of Depression." *Proceedings of the National Academy of Sciences of the United States of America* **111**, no. 1 (January 2014): 87–92.
  - Shayan Esfarayeni
    - Manicka, S., M. Marques-Pita, and L.M. Rocha [2022]. Effective connectivity determines the critical dynamics of biochemical networks. *Journal of the Royal Society Interface*. **19**(186):20210659.
  - Rik Pardun
    - Lindgren, K. [1991]."Evolutionary Phenomena in Simple Dynamics." In: *Artificial Life II*. Langton et al (Eds). Addison-wesley, pp. 295-312.
      - Optional: Hinton, G.E. and S.J. Nowlan [1987]."How learning can guide evolution." *Complex Systems*. **1**, pp.495-502.
  - Kaeli Ahn
    - Papadopoulou, Marina, Hanno Hildenbrandt, Daniel W. E. Sankey, Steven J. Portugal, and Charlotte K. Hemelrijk. "Self-Organization of Collective Escape in Pigeon Flocks." Edited by Feng Fu. *PLOS Computational Biology* **18**, no. 1 (January 10, 2022): e1009772.
- April 22, 2025
  - Rik Pardun
    - Conrad, M. [1990]. "The geometry of evolution." *Biosystems* **24**: 61-81.
  - Ngo Tuan Kiet
    - Garg, Shivam, Kirankumar Shiragur, Deborah M. Gordon, and Moses Charikar. "Distributed Algorithms from Arboreal Ants for the Shortest Path Problem."*Proceedings of the National Academy of Sciences* **120**, no. 6 (February 7, 2023): e2207959120.

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    - Garg, Shivam, Kirankumar Sh from Arboreal Ants for the Sh no. 6 (February 7, 2023): e22

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Syllabus

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Papers for Presentations

Papers for Presentations

Print | Settings

Add dates and restrictions...

Instructions for presentations:

Students are assigned to papers as lead discussants, but all students are supposed to read and participate in discussion of every paper. During class, a lead discussant prepares a short summary of the paper (15 minutes). The summary should:

1) Identify the key goals of the paper (not go in detail over every section);

2) What discussant liked and did not like;

3) What authors achieved and did not;

4) Any other relevant connections to other class materials and beyond.

After summary, discussion is opened to all, and role of lead discussant is to lead the discussion to make sure we address the important paper contributions. Also, discussant should prepare 2-3 discussion questions.

Upcoming Presentations:

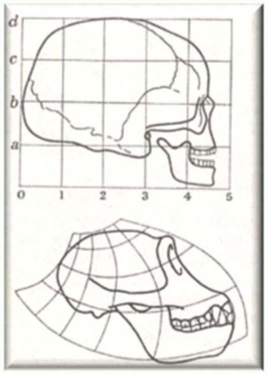
- *March 3rd, 2025*

  Emad Abed

  - Scheffer, Marten, et al. "Early-warning signals for critical transitions." *Nature* **461**.7260 (2009): 53.
    - Optional: Leemput, Ingrid A van de, Marieke Wichers, Angélique O J Cramer, Denny Borsboom, Francis Tuerlinckx, Peter Kuppens, Egbert H van Nes, et al. "Critical Slowing down as Early Warning for the Onset and Termination of

Additional information

- **Class Book**
  - Floreano, D. and C. Mattiussi [2008]. *Bio-Inspired Artificial Intelligence: Theories, Methods, and Technologies*. MIT Press. **Preface, Sections 4.1, 4.2**.
    - Nunes de Castro, Leandro [2006]. *Fundamentals of Natural Computing: Basic Concepts, Algorithms, and Applications*. Chapman & Hall. **Chapter 1,** pp. 1-23. Chapter 7, sections **7.1, 7.2** and **7.4** – Fractals and L-Systems, **Appendix B.3.1** – Production Grammars
- **Lecture notes**
  - Chapter 1: What is Life?
  - Chapter 2: The logical Mechanisms of Life
  - Chapter 3: Formalizing and Modeling the World
    - posted online @ casci.binghamton.edu/academics/i-bic



- **Papers and other materials**
  - Optional
    - Wigner, E.P. [1960], "The unreasonable effectiveness of mathematics in the natural sciences". *Comm. Pure Appl. Math.*, **13**: 1-14.
    - Flake's [1998], *The Computational Beauty of Life*. MIT Press.
      - Chapter **1** – Introduction
      - Chapters **5, 6** (7-9) – Self-similarity, fractals, L-Systems

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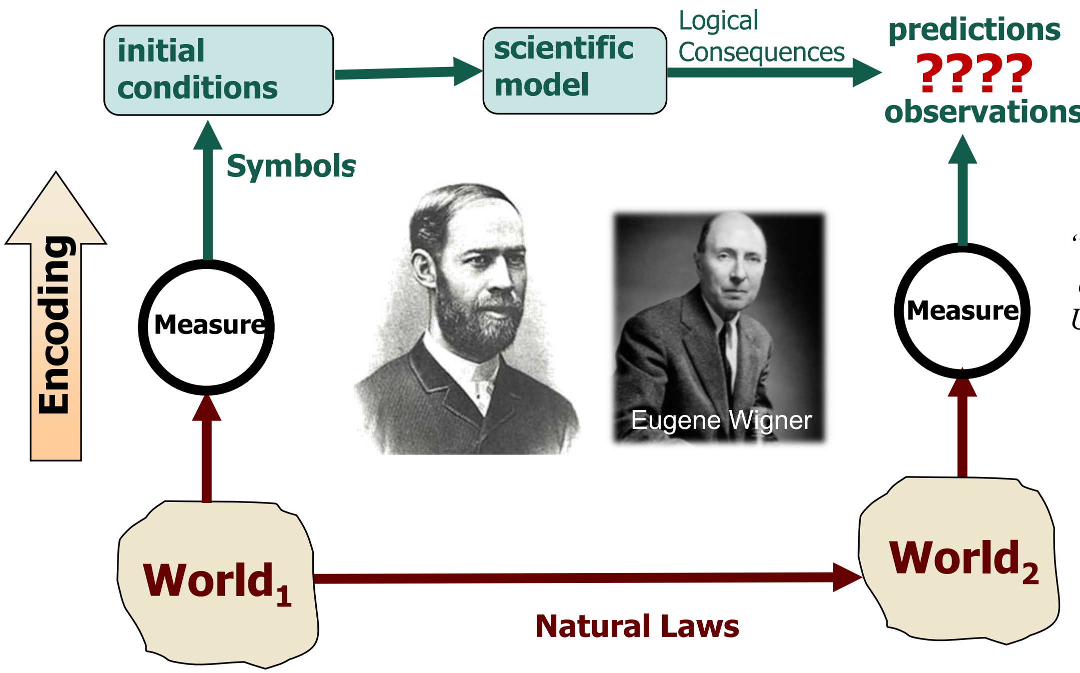
Hertzian scientific modeling paradigm



*"**Every empirical law has the disquieting quality that one does not know its limitations**." E. Wigner [1957] in "The Unreasonable Effectiveness of Mathematics in the Natural Sciences"*

"The most direct and in a sense the most important problem which our conscious knowledge of nature should enable us to solve is the ***anticipation of future events***, so that we may arrange our present affairs in accordance with such anticipation". (Hertz, 1894)

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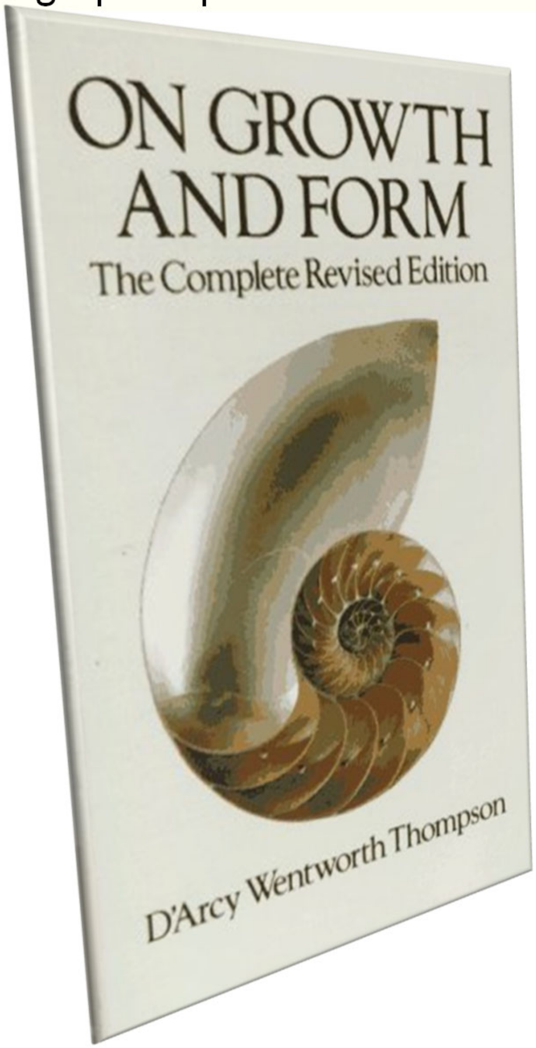
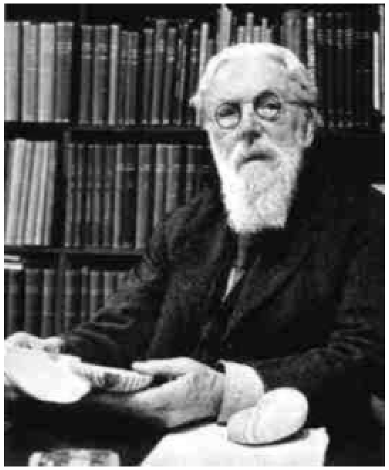
design principles



- D'Arcy Wentworth Thompson (1860 - 1948)
  - *On Growth and Form* (1917), laid the foundations of bio-mathematics
    - Equations to describe static patterns of living organisms
      - Shells, cauliflower head, etc.
    - Transformations of form changing a few parameters



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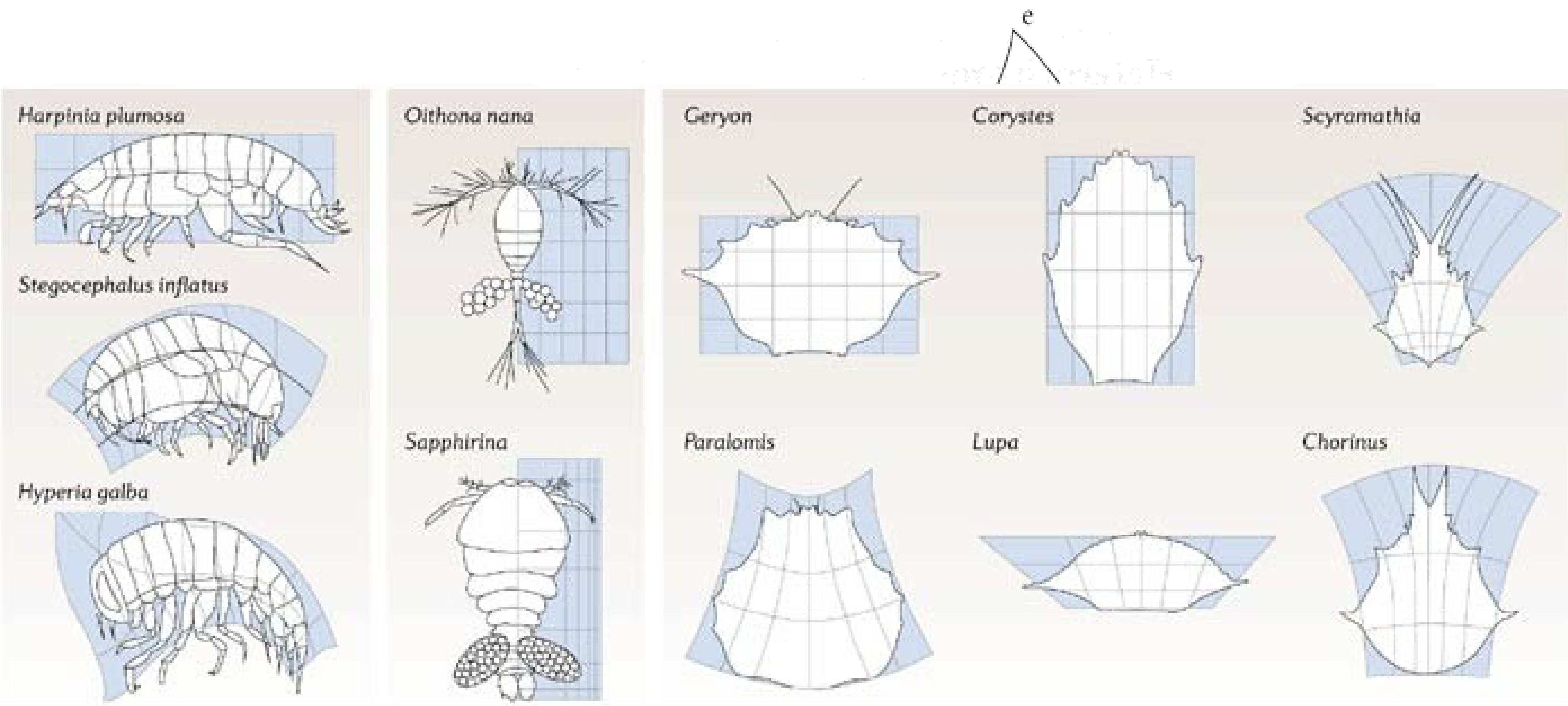
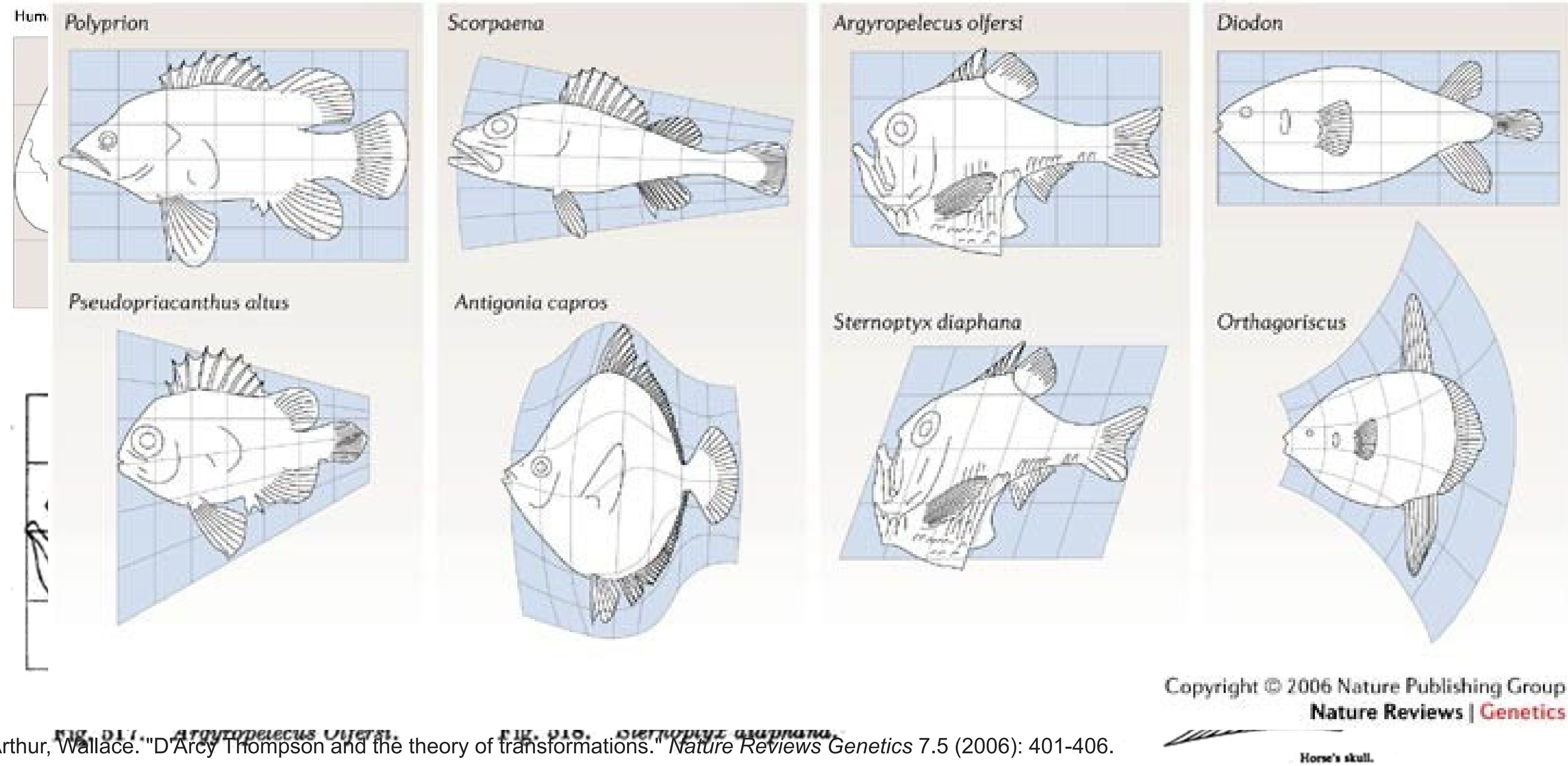


Nature Reviews | Genetics

Arthur, Wallace. "D'Arcy Thompson and the theory of transformations." *Nature Reviews Genetics* 7.5 (2006): 401-406.

D'Arcy Thompson



Arthur, Wallace. "D'Arcy Thompson and the theory of transformations." *Nature Reviews Genetics* 7.5 (2006): 401-406.

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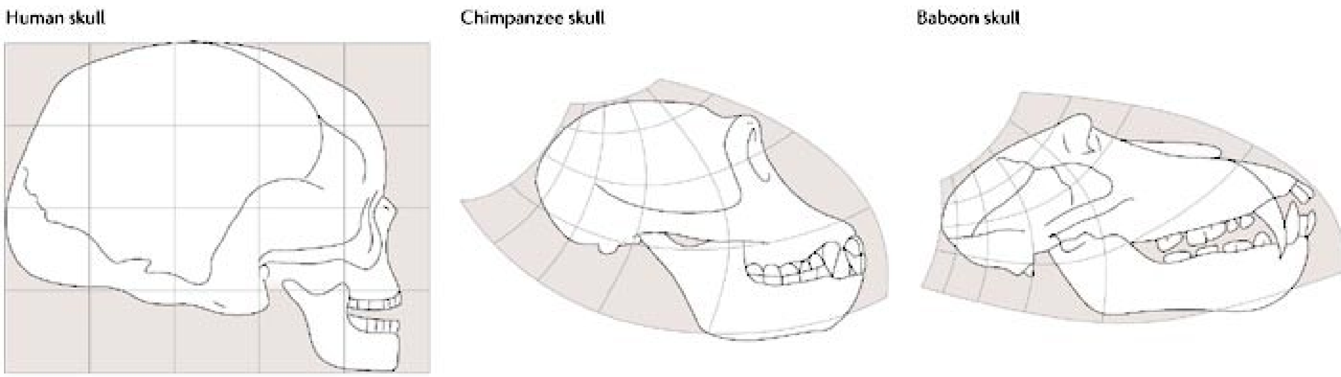
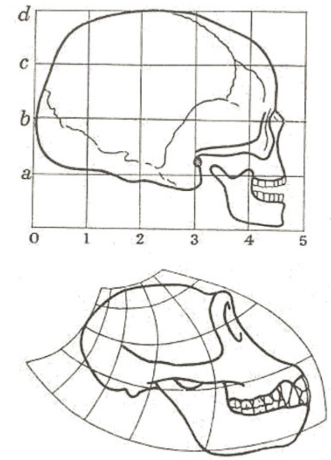
D'Arcy Thompson





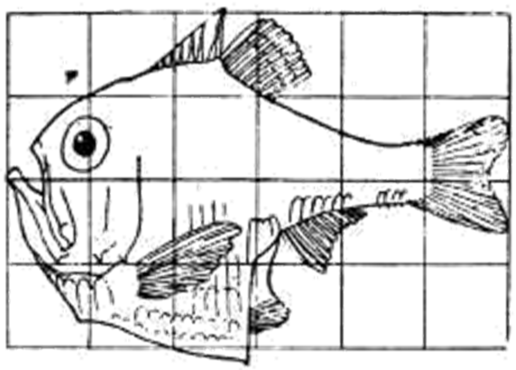


Fig. 517. *Argyropelecus Olfersi.*

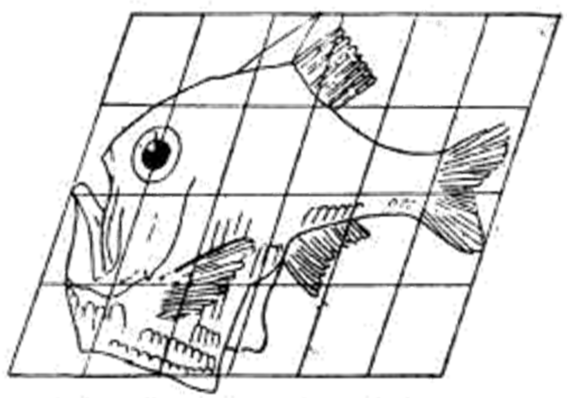
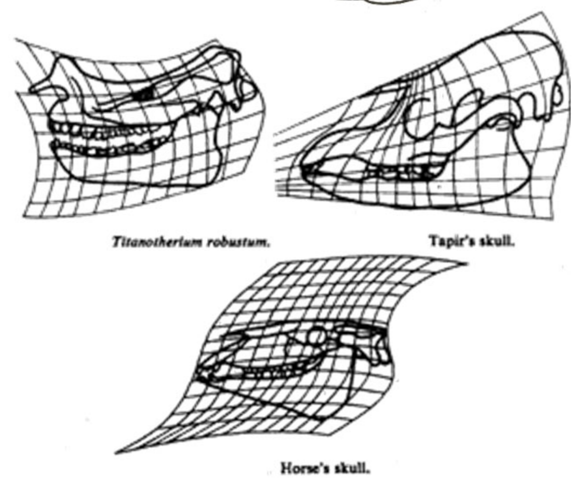


Fig. 518. *Sternoptyx diaphana.*



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life inspired

SSIE-483X/583X: evolutionary systems and biologically inspired computing

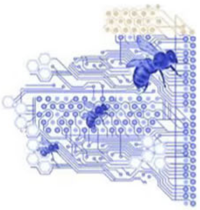
lifelines
Luis M. Rocha on Twitter
i-bic
SSIE
BU Calendar

archives

Sunday, February 09, 2025

Lecture Notes (Chapter 2): The Logical Mechanisms of Life

The Logical Mechanisms of Life



Labels: #ArtificialLife, #BiologicalTheory, #ComplexSystems, #GeneralSystems, #SystemsScience, #Theory

posted by Luis Rocha @ 6:19 PM 0 comments

Tuesday, January 28, 2025

Self-Assembling Wires

The video shown in class of self-assembly of wires made of metal bearings.

modeling similarities across nature

- self-similar structures
  - Trees, plants, clouds, mountains
    - morphogenesis
  - Mechanism
    - Iteration, recursion, feedback
- dynamical systems and unpredictability
  - From limited knowledge or inherent in nature?
  - Mechanism
    - Chaos, measurement
- self-organization, collective behavior, emergence
  - Complex behavior from collectives of many simple units or agents
    - cellular automata, dynamical networks, morphogenesis, swarms, brains, social systems
  - Mechanism
    - Parallelism, multiplicity, multi-solutions, redundancy
- evolution
  - Adaptation, learning, social evolution
  - Mechanism
    - Reproduction, transmission, variation, selection, Turing's tape
- Network causality (heterogenous complexity)
  - Behavior derived from many inseparable sources
    - Immune system, anticipatory systems, brain-body-environment-culture, embodiment, epigenetics, culture
  - Mechanism
    - Modularity, control, hierarchy, connectivity, stigmergy, redundancy

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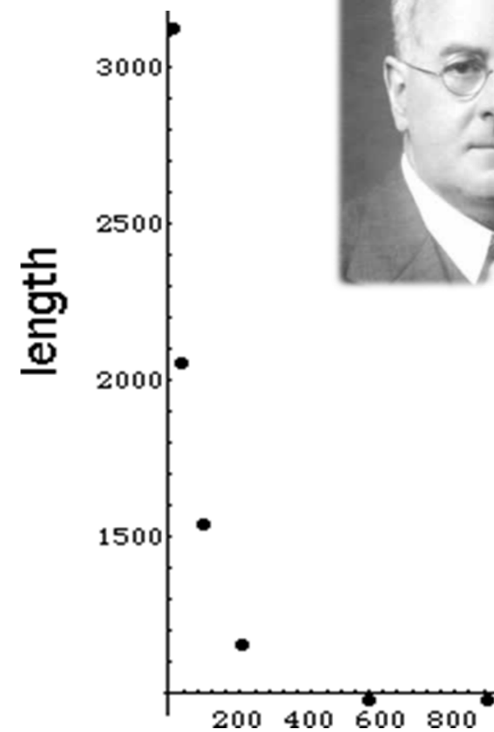
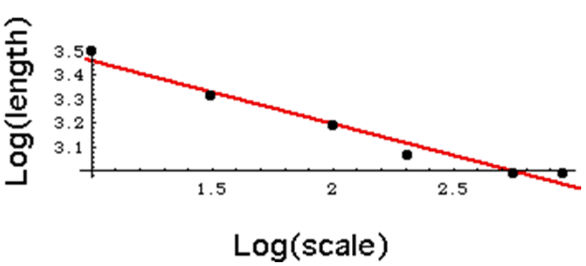
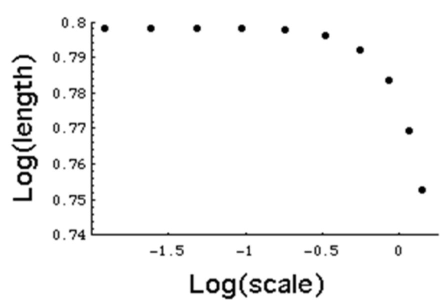
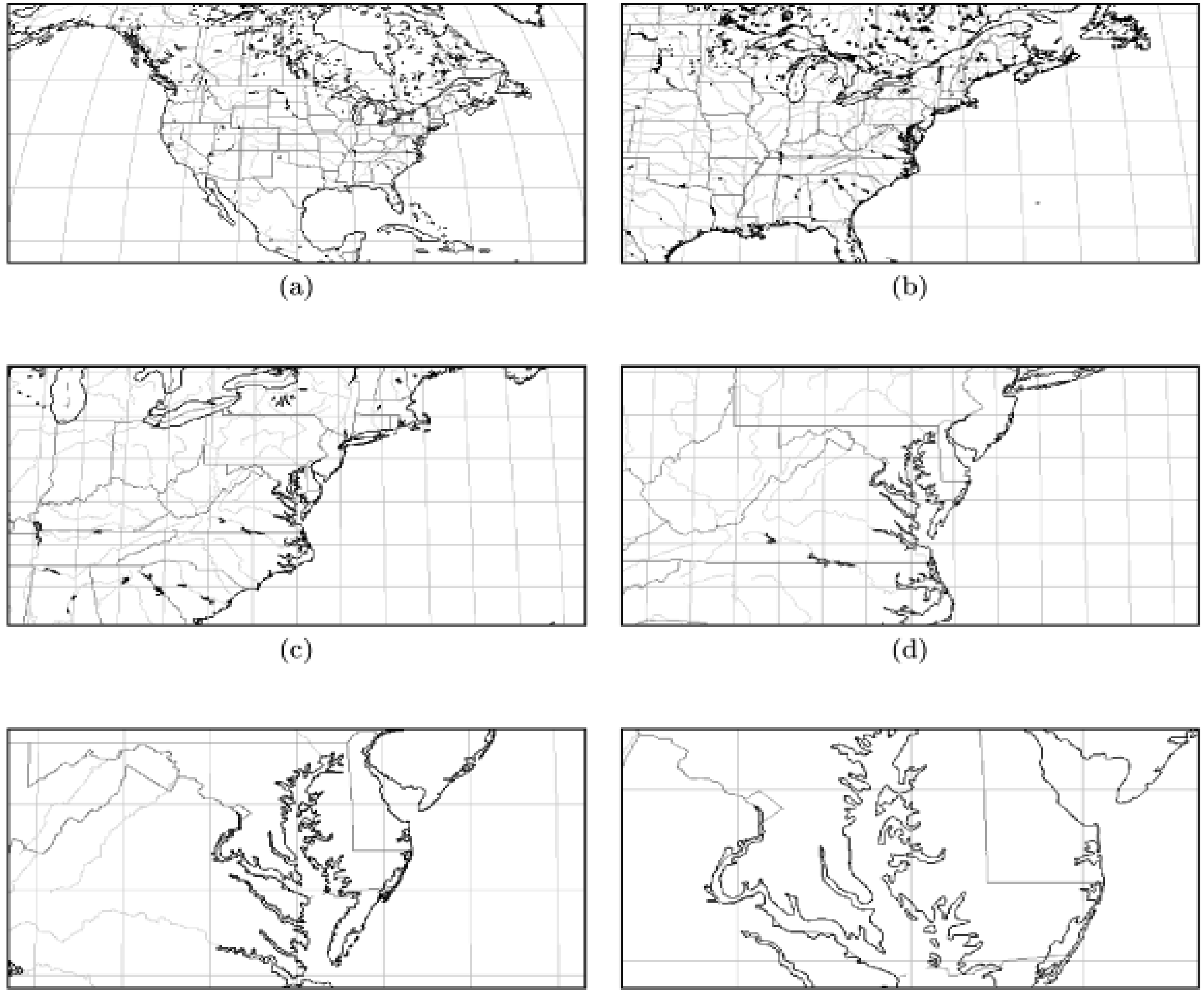
Lewis Richardson's observations (1961)

- Measured maps with different scales
  - Coasts of Australia, South Africa, and Britain
  - Land frontiers of Germany and Portugal
  - Measured lengths *L*(*d*) at different scales *d*.
    - As the scale is reduced, the length increases rapidly.
  - Well-fit by a straight line with slopes (s) on log/log plots
    - s = -0.25 for the west coast of Britain, one of the roughest in the atlas,
    - s = -0.15 for the land frontier of Germany,
    - s = -0.14 for the land frontier of Portugal,
    - s = -0.02 for the South African coast, one of the smoothest in the atlas.
    - circles and other smooth curves have line of slope 0.



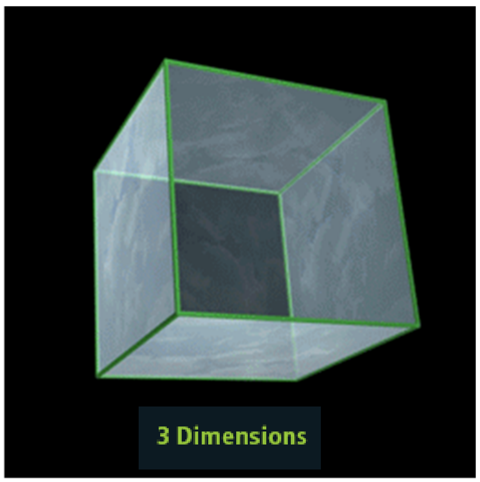


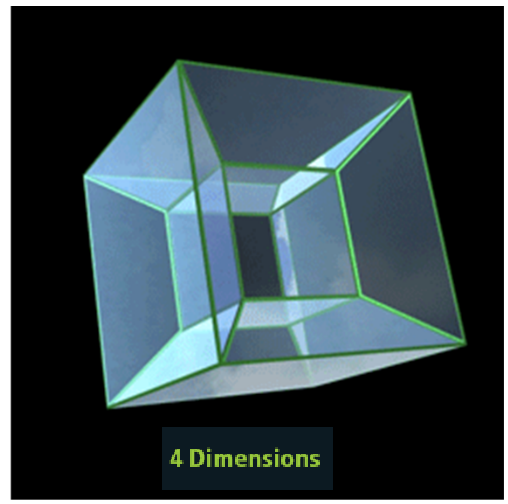




(a) (b) (c) (d) (e) (f)

Integer dimensions

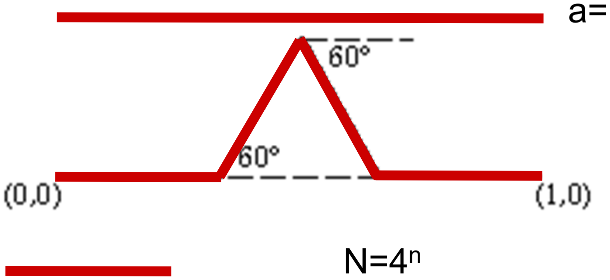




Scientific American, July 2008

the Koch curve example: fractional dimensions

- **Koch curve**
  - slightly more than line but less than a plane
  - Packing efficiency!



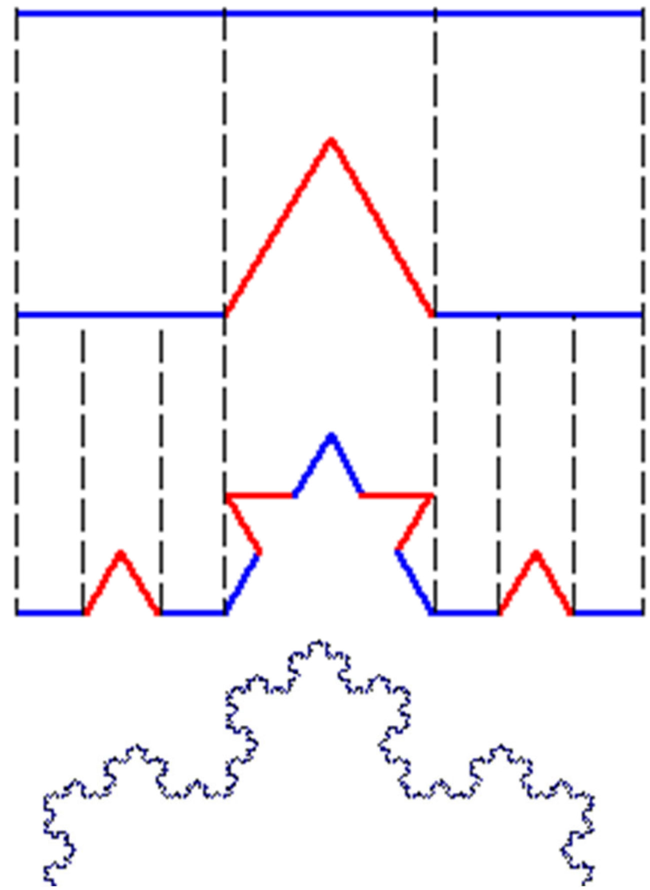
$$D = \frac{\log N}{\log\left(\frac{1}{a}\right)} = \frac{\log 4}{\log 3} = 1.26186...$$

Hausdorff Dimension

Number of units

$$N = \left(\frac{1}{a}\right)^D \Rightarrow D = \frac{\log N}{\log\left(\frac{1}{a}\right)}$$

Unit measure



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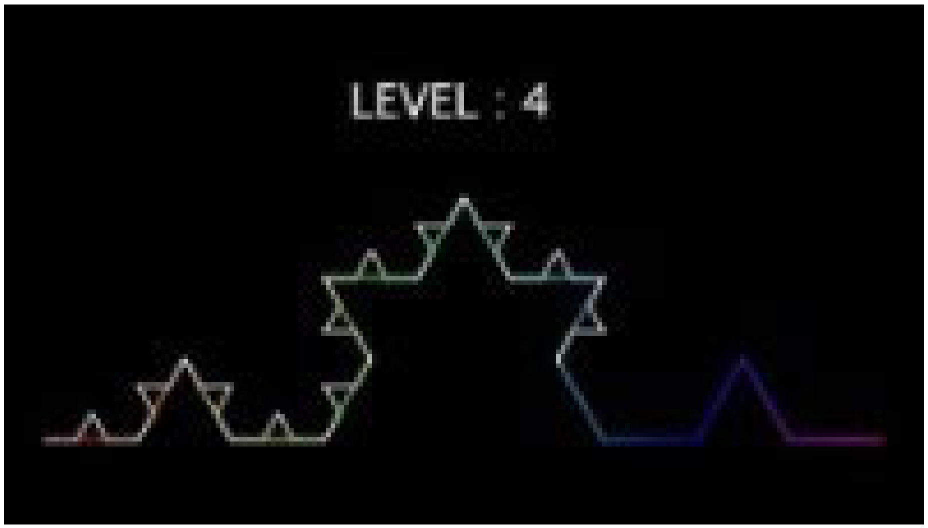
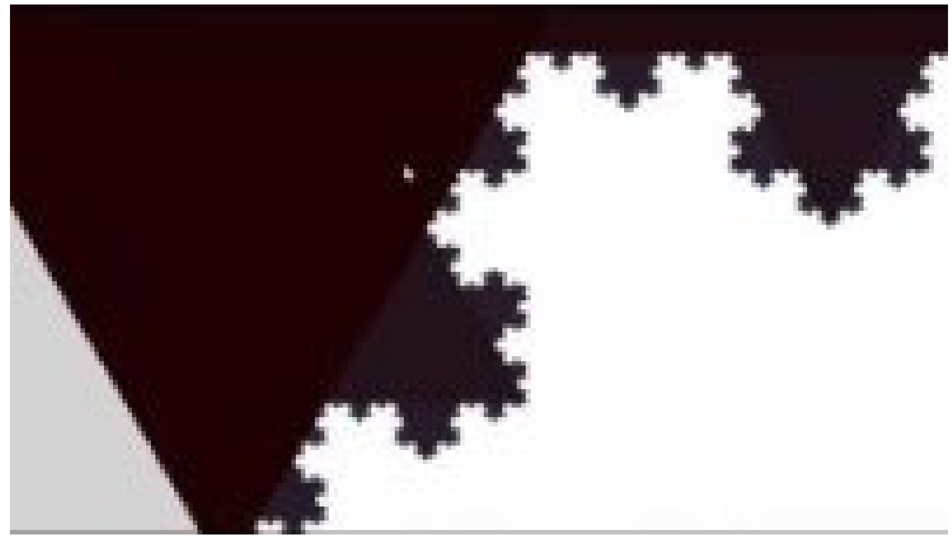
videos





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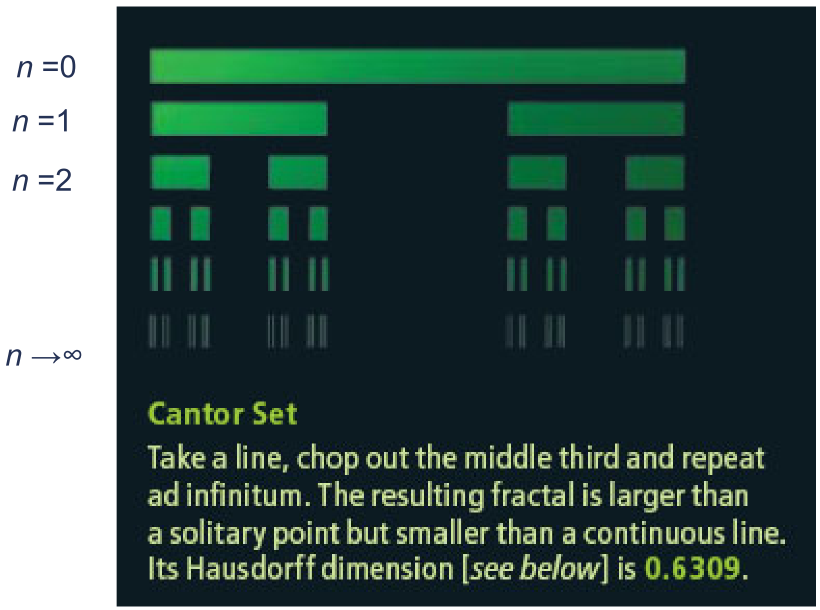
mathematical monsters

- Complex objects are defined by systematically and recursively replacing parts of a simple start object with another object according to a simple rule
  - Cantor Set



Scientific American, July 2008

$$D = \frac{\log N}{\log\left(\frac{1}{a}\right)} = 0.6309$$

Hausdorff Dimension

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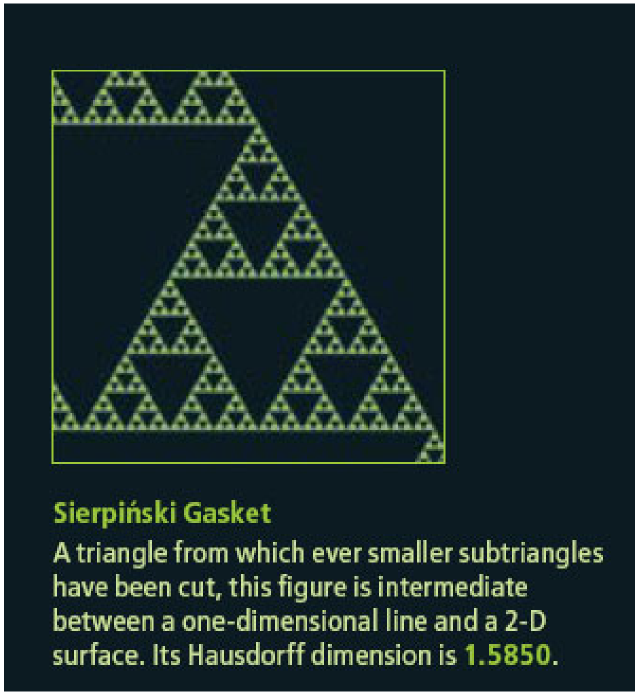
mathematical monsters

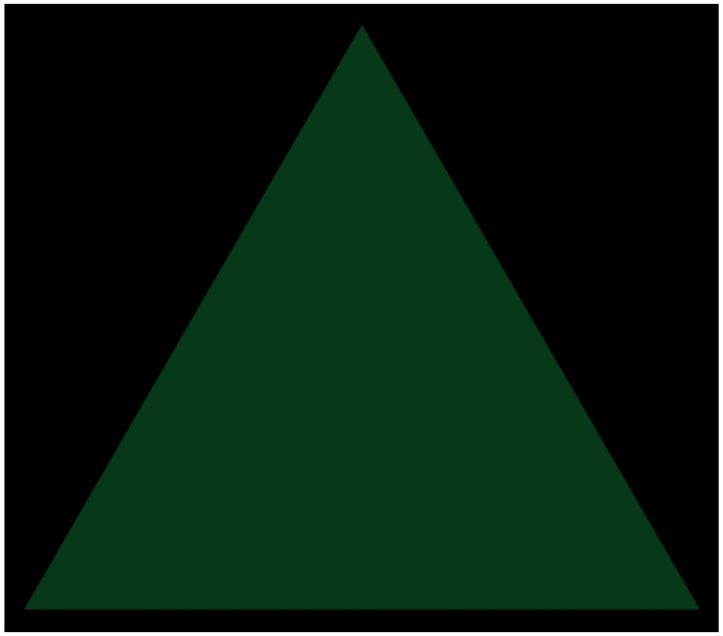
- Complex objects are defined by systematically and recursively replacing parts of a simple start object with another object according to a simple rule
  - Sierpinski Gasket



Scientific American, July 2008

$$D = \frac{\log N}{\log\left(\frac{1}{a}\right)} = 1.585$$

Hausdorff Dimension



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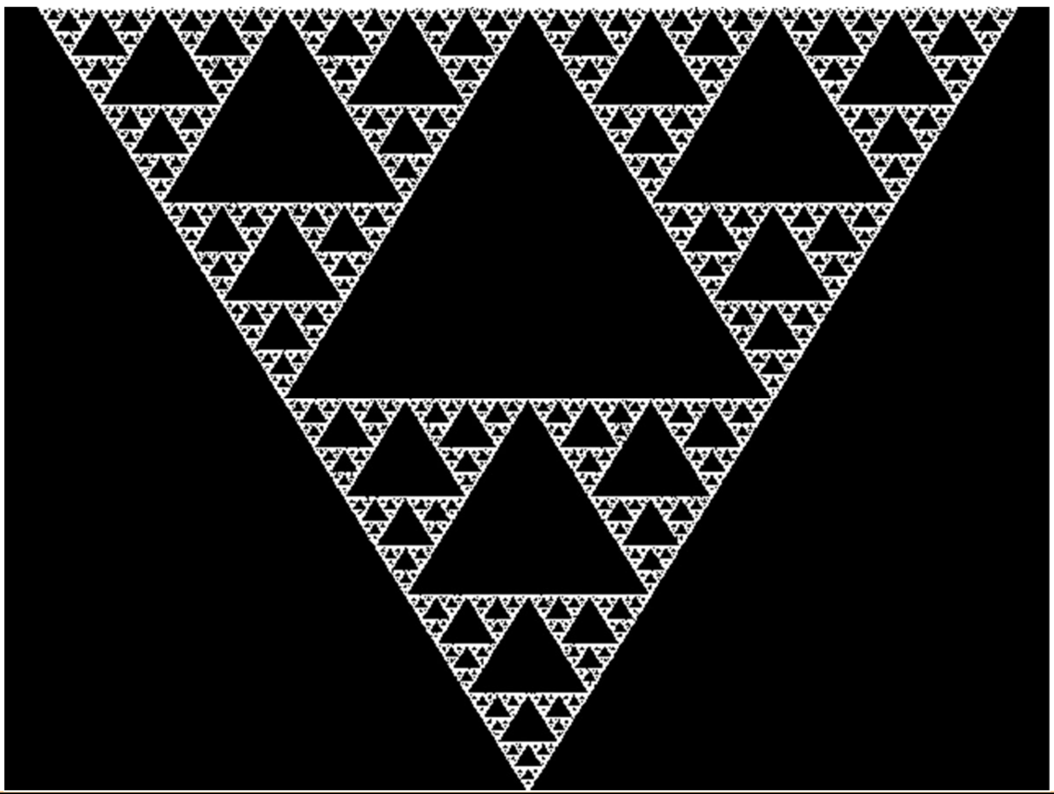
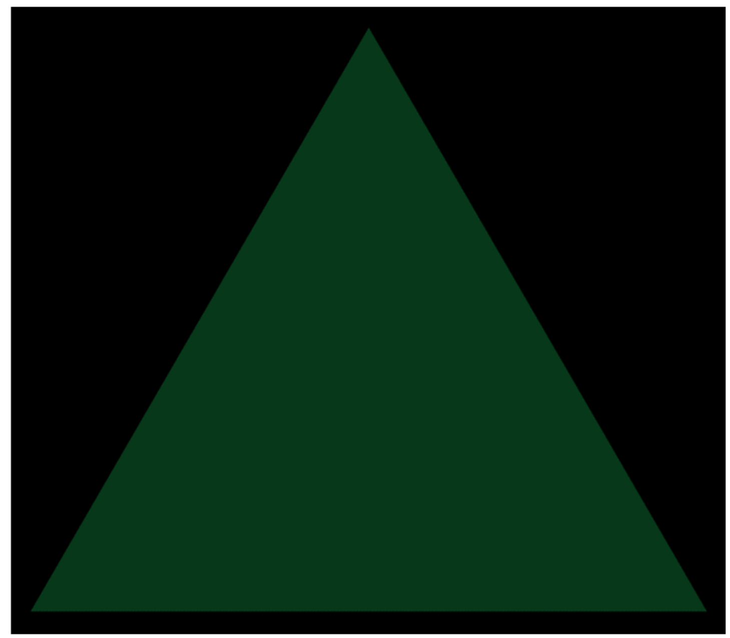
mathematical monsters

- Complex objects are defined by systematically and recursively replacing parts of a simple start object with another object according to a simple rule
  - Sierpinski Gasket





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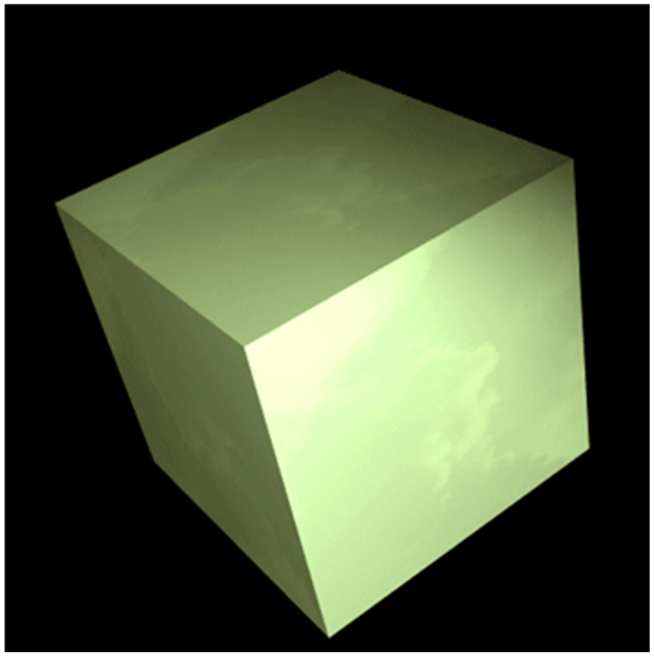
mathematical monsters

- Complex objects are defined by systematically and recursively replacing parts of a simple start object with another object according to a simple rule
  - Menger sponge

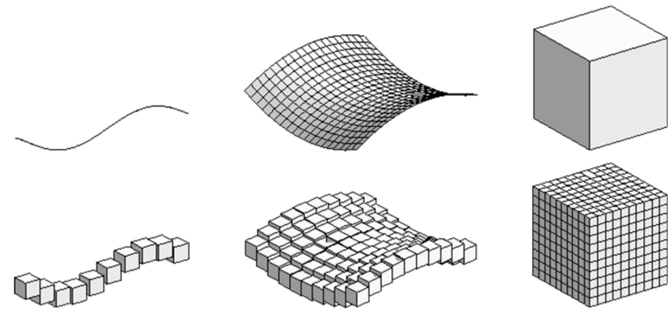
Scientific American, July 2008

$$D = \frac{\log N}{\log\left(\frac{1}{a}\right)} = 2.7268$$

Hausdorff Dimension



Box-counting dimension



Number of boxes

$$D = \lim_{\varepsilon \to 0} \frac{\log N(\varepsilon)}{\log\left(\frac{1}{\varepsilon}\right)}$$

Length of box side

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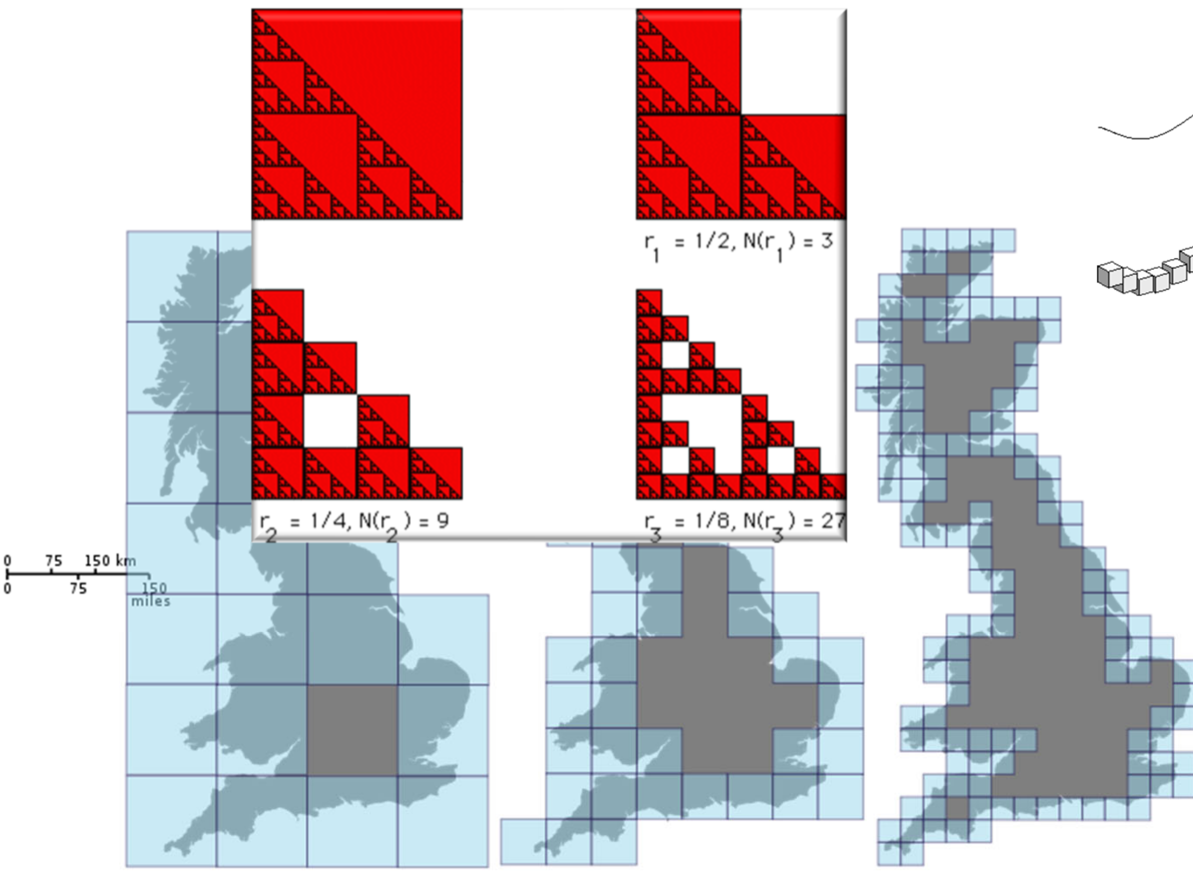
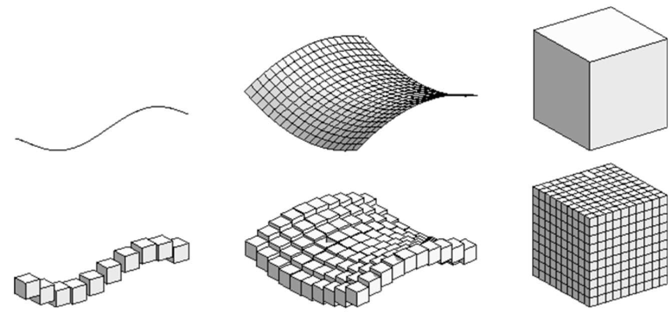
Box-counting dimension

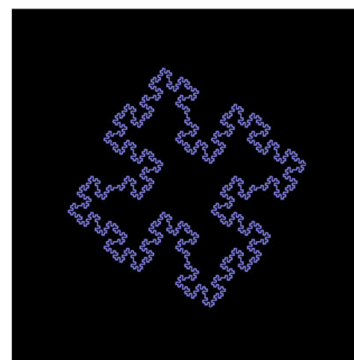




Number of boxes

$$D = \lim_{\varepsilon \to 0} \frac{\log N(\varepsilon)}{\log\left(\frac{1}{\varepsilon}\right)}$$

Length of box side



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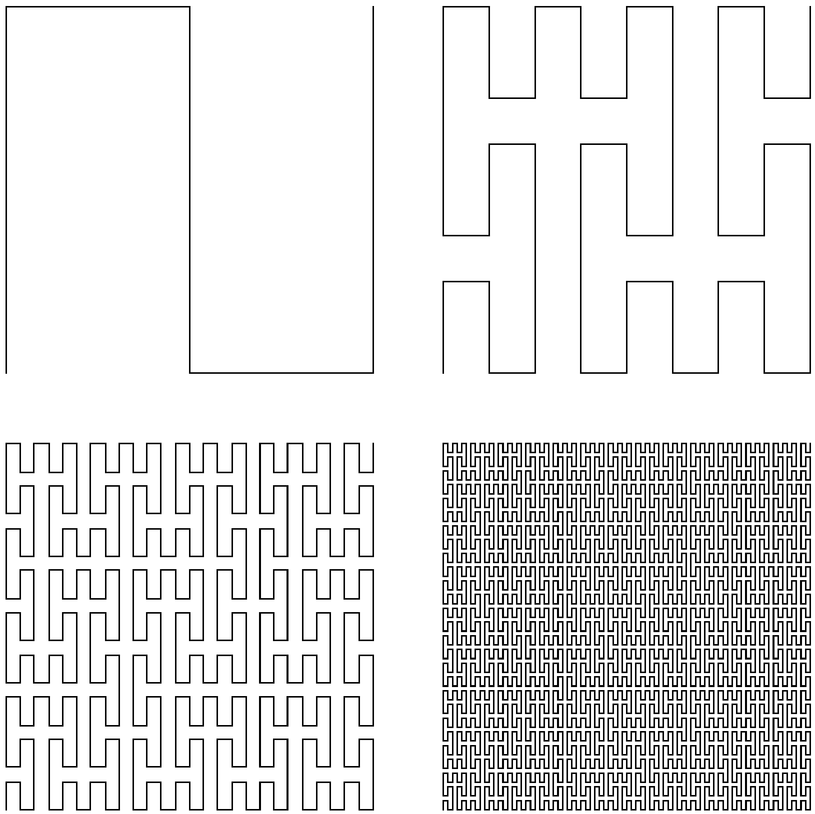
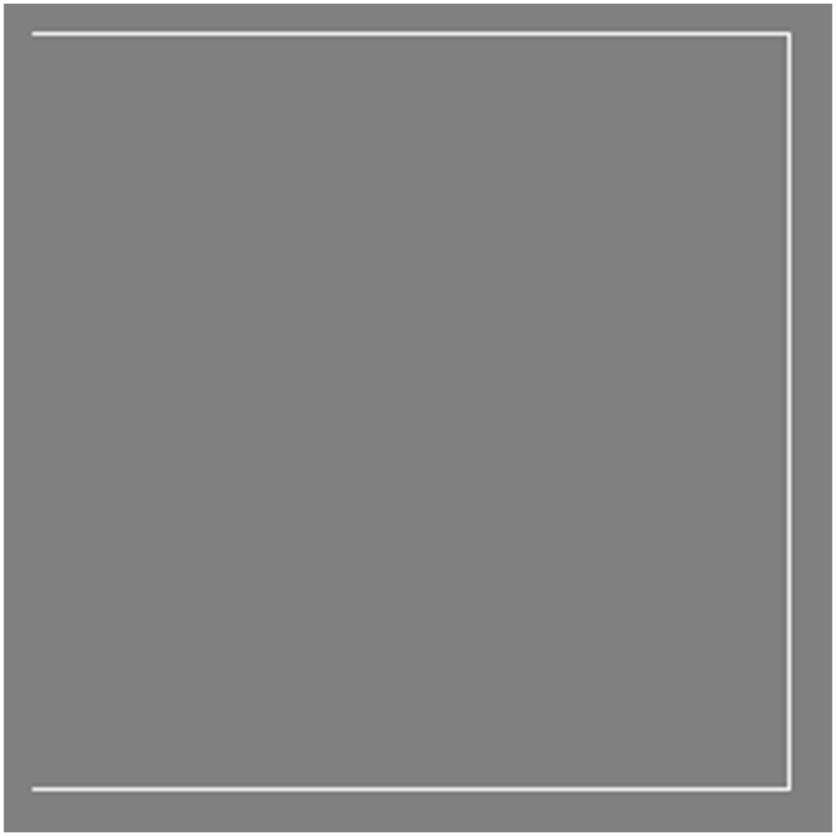
Filling planes and volumes

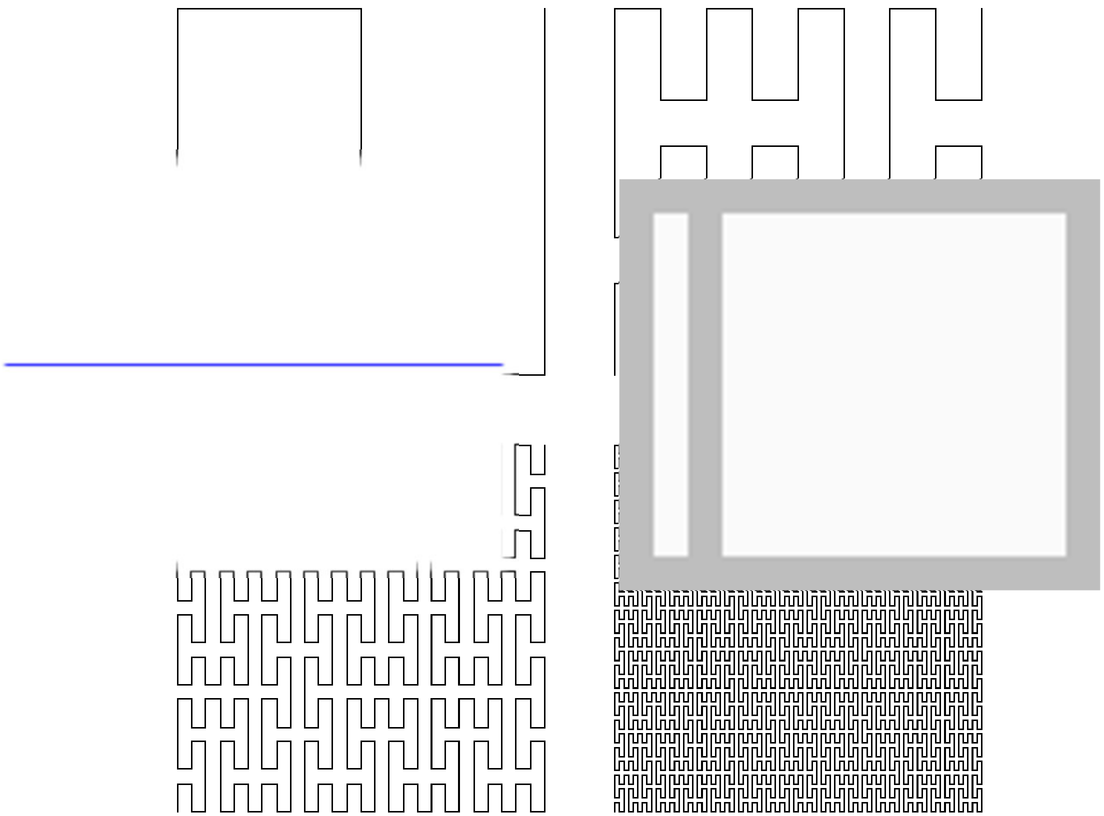


Peano

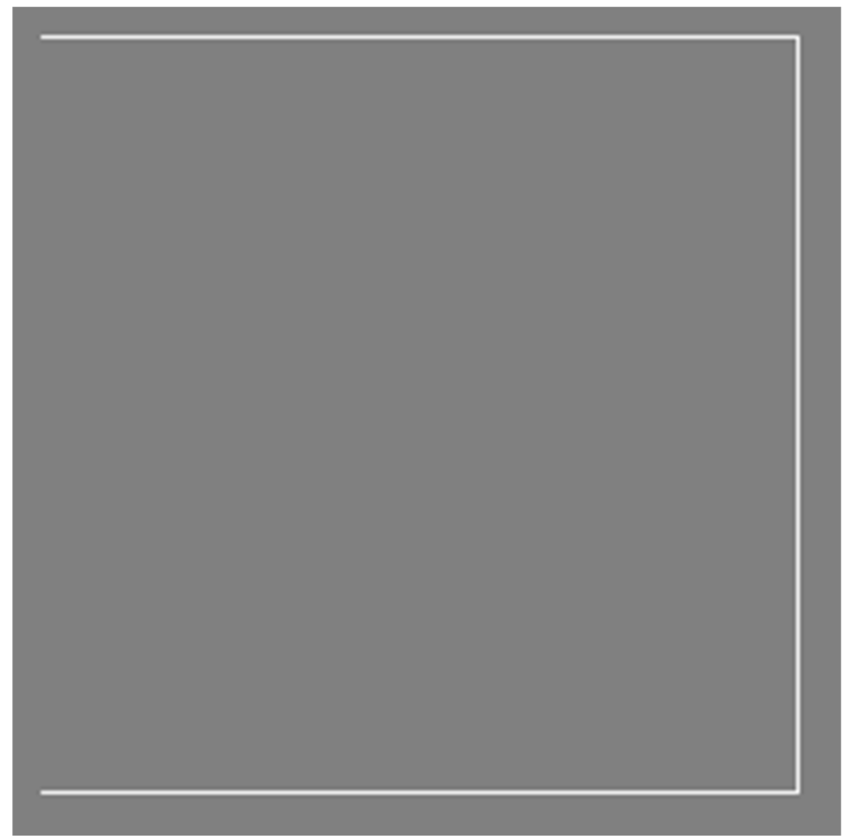


Hilbert

Filling planes and volumes



Peano



Hilbert

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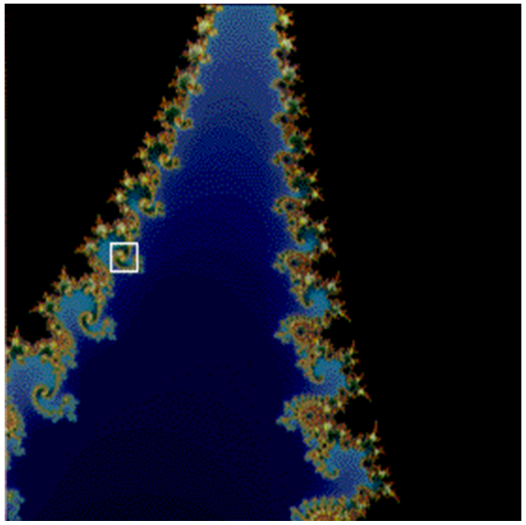
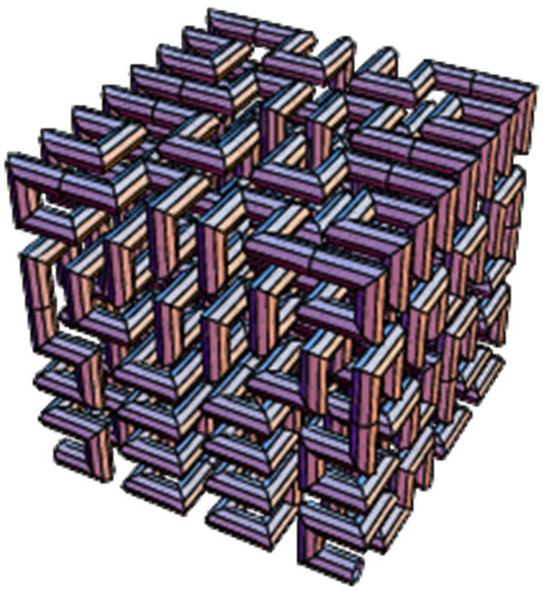
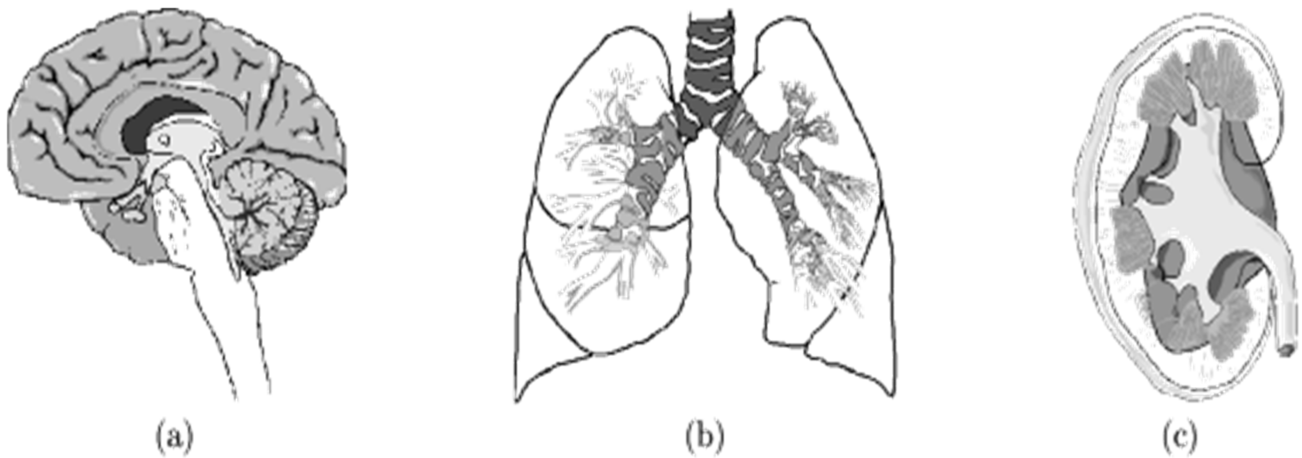
- Self-similarity on multiple scales
  - Due to recursion
- Fractal dimension
  - Enclosed in a given space, but with infinite number of points or measurement



reducing volume





How do these packed volumes and recursive morphologies grow?

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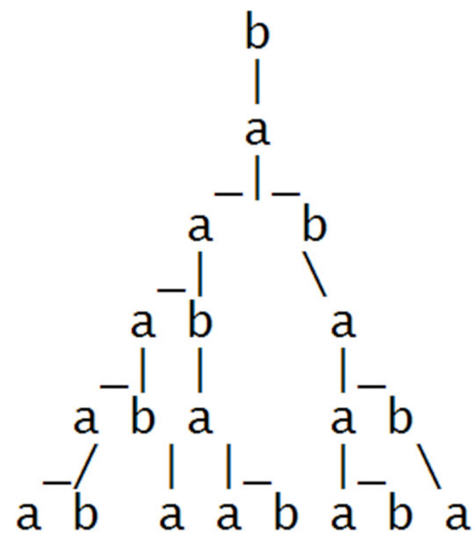
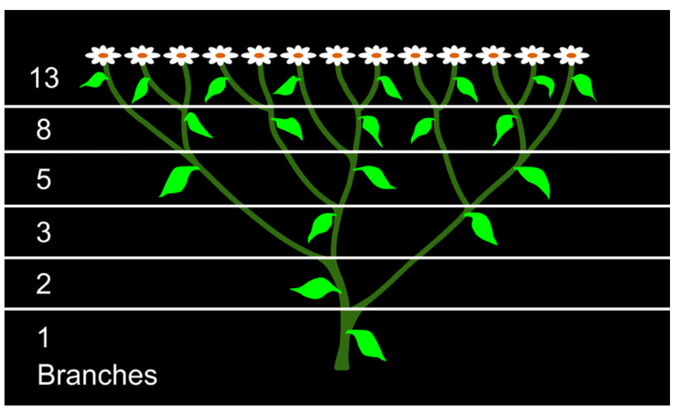
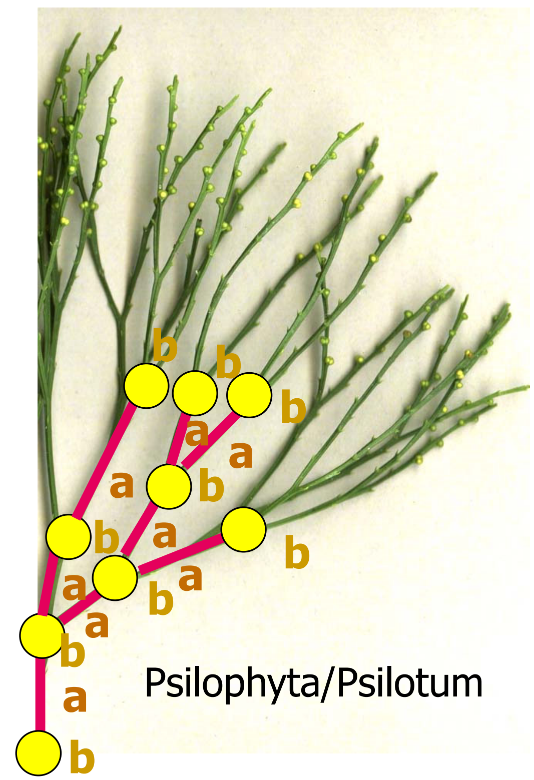
branching as a general system

- An Accurate Model
  - Requires
    - Varying angles
    - Varying stem lengths
    - randomness
  - The Fibonacci Model is similar
    - Initial State: b
    - b -> a
    - a -> ab
  - *sneezewort*







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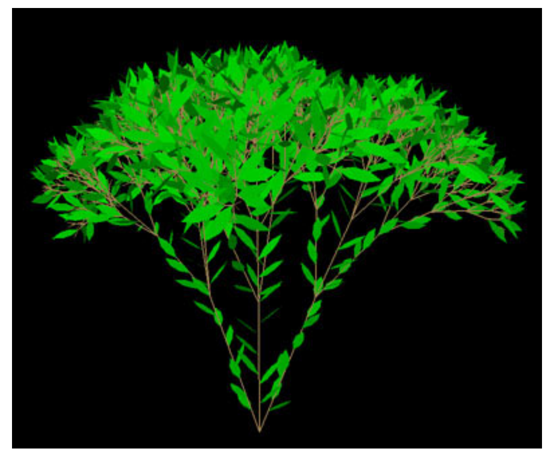
Aristid Lindenmeyer

- Mathematical formalism proposed by the biologist Aristid Lindenmayer in 1968 as a foundation for an axiomatic theory of biological development.
  - applications in computer graphics
    - Generation of fractals and realistic modeling of plants
  - Grammar for rewriting Symbols
    - Production Grammar
    - Defines complex objects by successively replacing parts of a simple object using a set of recursive, rewriting rules or productions.
      - Beyond one-dimensional production (Chomsky) grammars
      - Parallel ***recursion***
      - Access to computers



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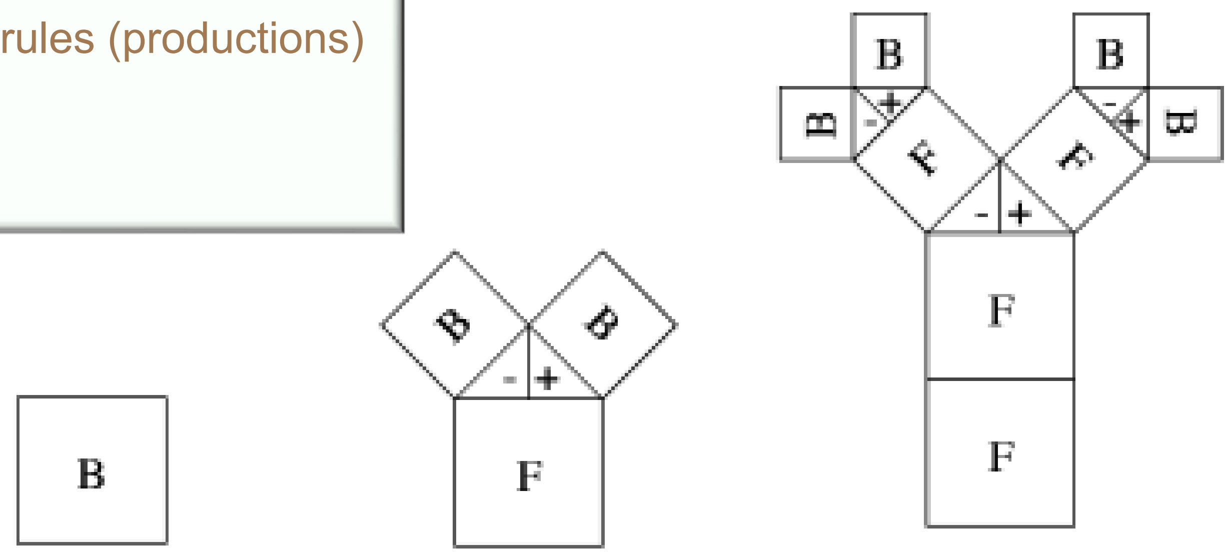
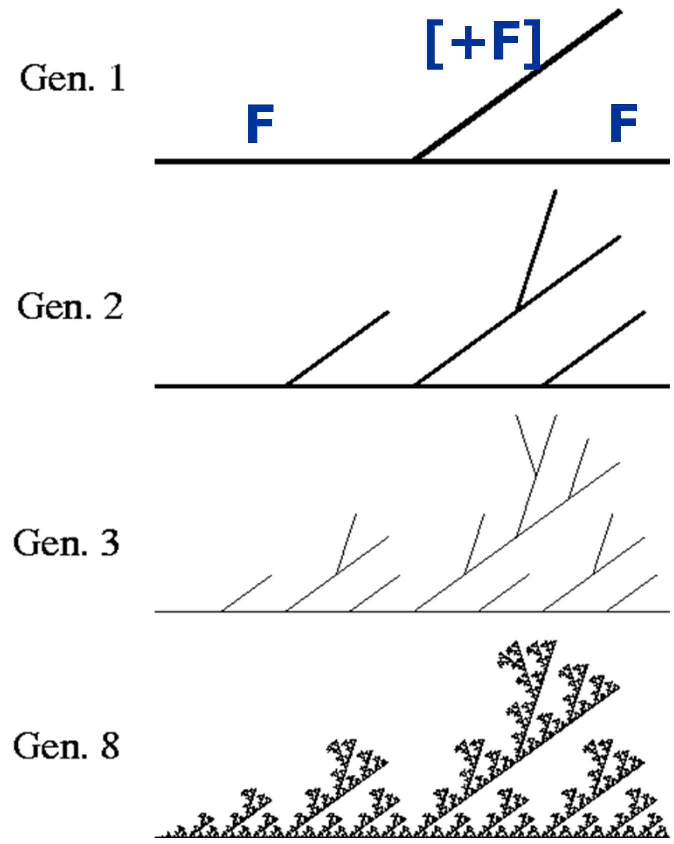
formal notation of the production system

- An L-system is an ordered triplet
  - $G = <V, w, P>$
    - V = alphabet of the symbols in the system
      - V = {F, B}
    - w = nonempty word
      - the axiom: B
    - P = finite set of production rules (productions)
      - B → F[-B][+B]
      - F → FF



production rules for artificial plants

- Add branching symbols [ ]
  - Main trunk shoots off one side branch
  - simple example
    - Angle 45
    - Axiom: F
      - Seed Cell
    - Rule: F=F[+F]F
- Deterministic, context-free L-systems
  - Simplest class of L-systems
  - Simple re-writing
  - D0L



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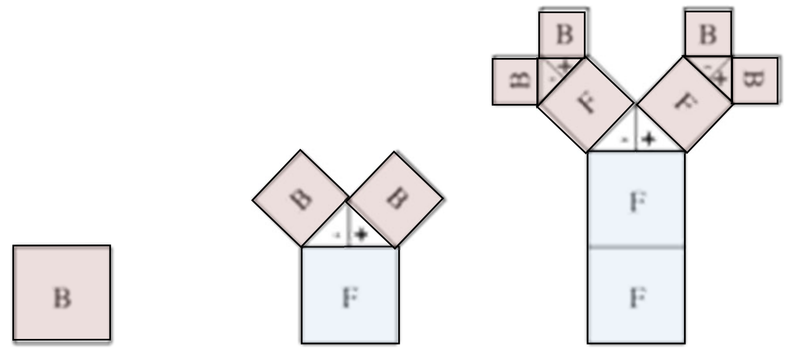
- Axiom
  - B
- Cell Types
  - B,F
- Rules
  - B → F[-B][+B]
  - F → FF



Depth	Resulting String
0	B
1	F[-B][+B]
2	FF[-F[-B][+B]]+[+F[-B][+B]]
3	FFFF[-FF[-F[-B][+B]][+F[-B][+B]]]+[FF[-F[-B][+B]][+F[-B][+B]]]

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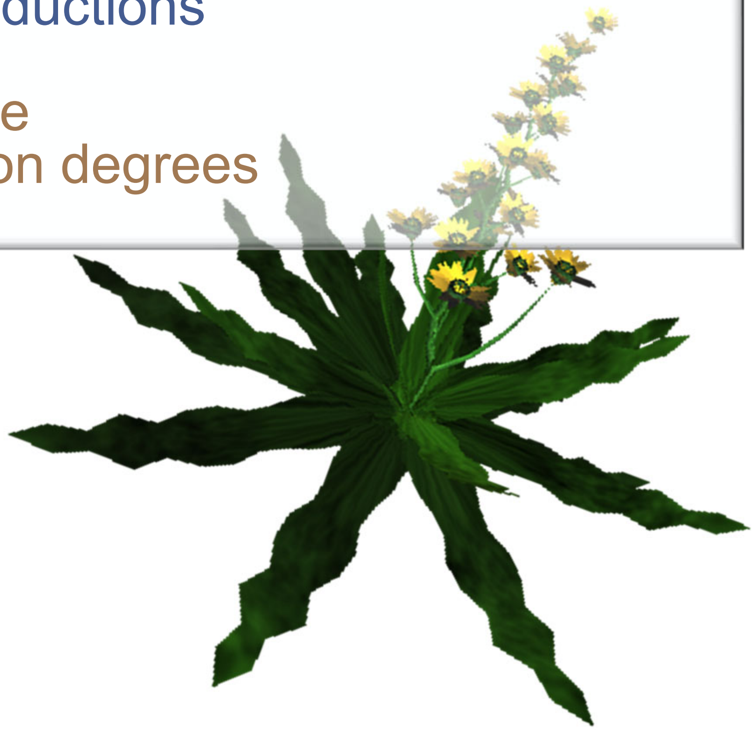
- Discrete nature of L-systems makes it difficult to model continuous phenomena
  - Numerical parameters can be associated with L-system symbols
  - Parameters control the effect of productions
  - A (t) → B (t x 3)
    - Growth can be modulated by time
    - Varying length of braches, rotation degrees



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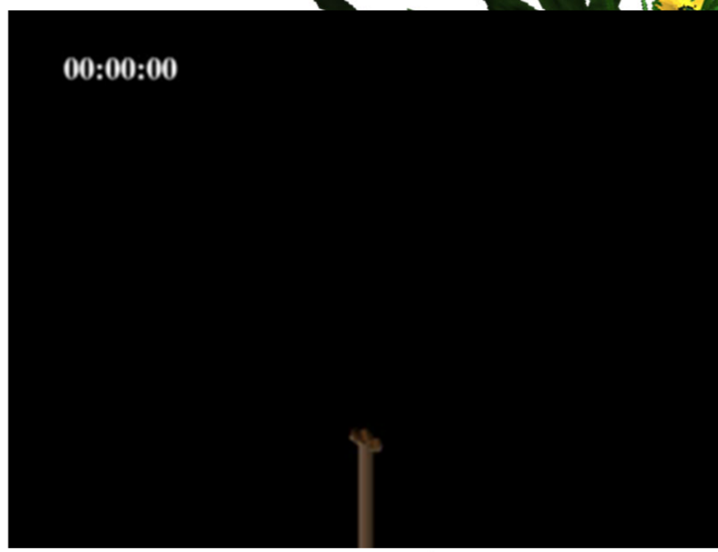
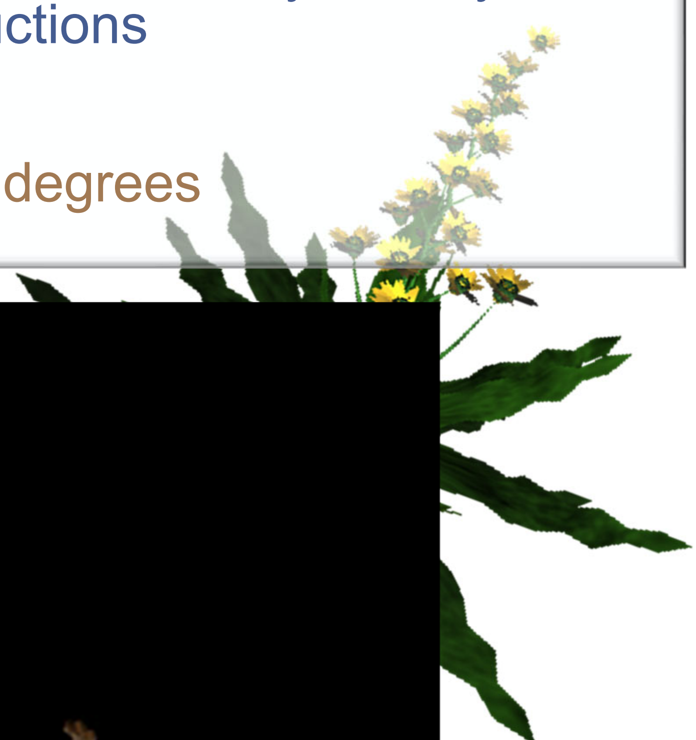
parametric L-systems

- Discrete nature of L-systems makes it difficult to model continuous phenomena
  - Numerical parameters can be associated with L-system symbols
  - Parameters control the effect of productions
  - A (t) → B (t x 3)
    - Growth can be modulated by time
    - Varying length of braches, rotation degrees





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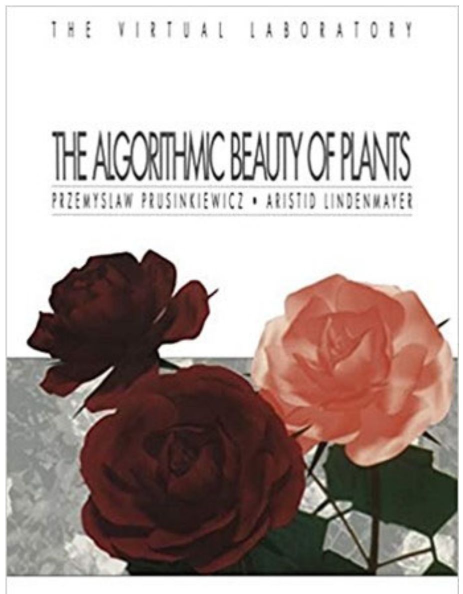
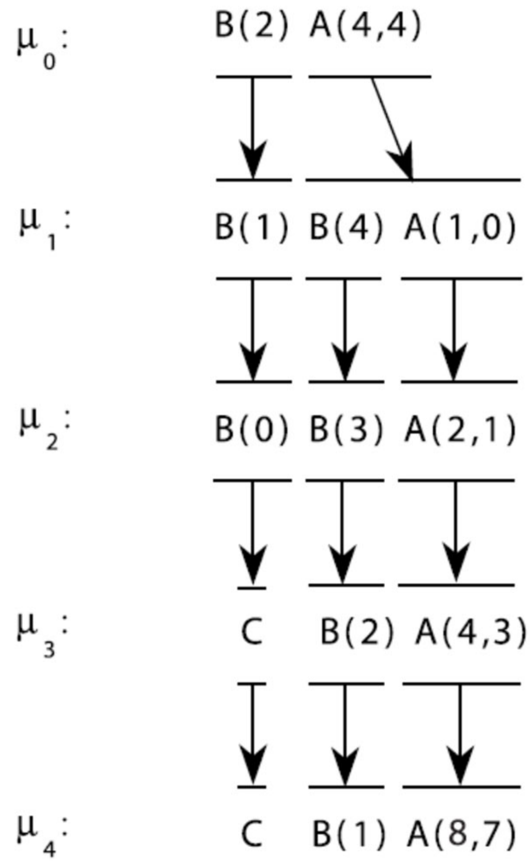
example

$$
\begin{array}{llllll}
\omega & : & B(2)A(4,4) & & & \\
p_1 & : & A(x,y) & : y <= 3 & \rightarrow & A(x*2, x+y) \\
p_2 & : & A(x,y) & : y > 3 & \rightarrow & B(x)A(x/y, 0) \\
p_3 & : & B(x) & : x < 1 & \rightarrow & C \\
p_4 & : & B(x) & : x >= 1 & \rightarrow & B(x-1)
\end{array}
$$

operate on ***parametric words***, which are strings of modules consisting of symbols with associated parameters and their rules

From: P. Prusinkiewicz and A. Lindenmayer [1991]. *The Algorithmic Beauty of Plants*.





example

$$
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operate on ***parametric words***, which are strings of modules consisting of symbols with associated parameters and their rules

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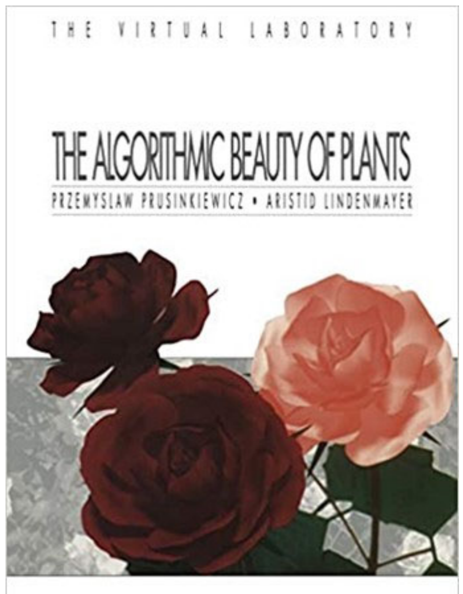
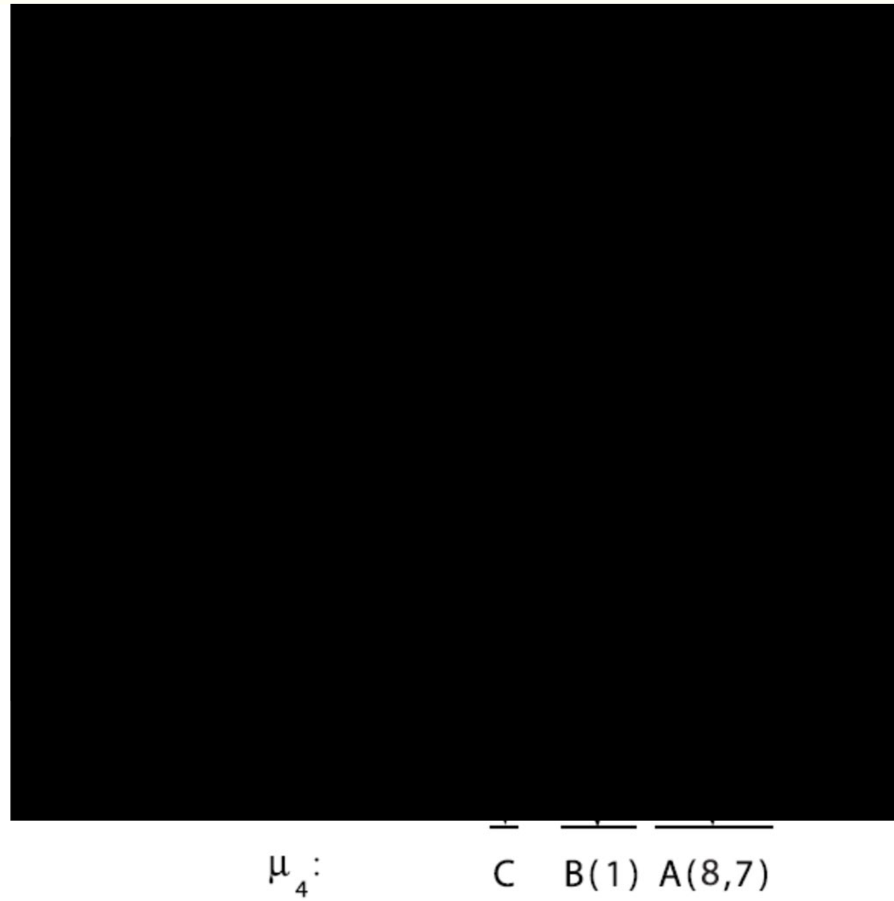
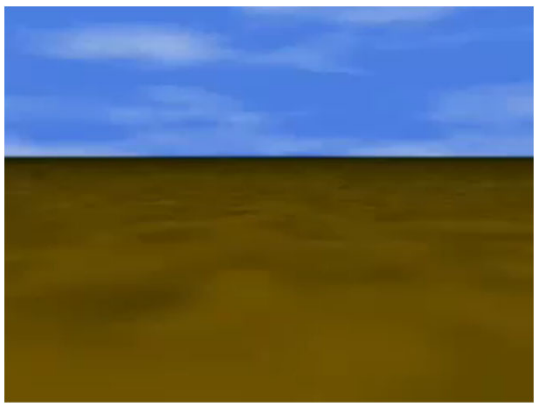
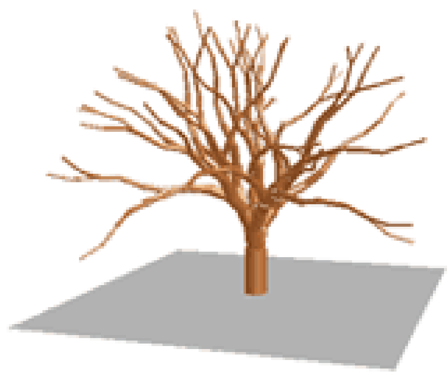


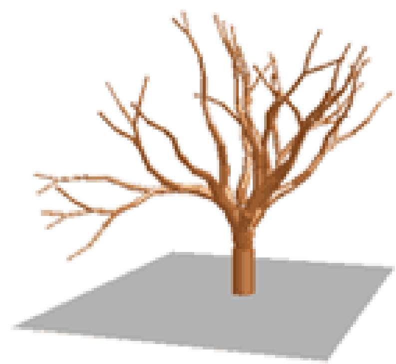


stochastic L-systems

- Probabilistic production rules
  - A → B C (P = 0.3)
  - A → F A (P = 0.5)
  - A → A B (P = 0.2)







http://coco.ccu.uniovi.es/malva/sketchbook/

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2L-Systems

- Production rules depend on neighbor symbols in input string
  - simulates interaction between different parts
    - necessary to model information exchange between neighboring components
- 2L-Systems
  - $P: a_l < a > a_r \rightarrow X$
    - P1: A<F>A → A
    - P2: A<F>F → F
- 1L-Systems
  - $P: a_l < a \rightarrow X$ or $P: a > a_r \rightarrow X$
- Generalized to IL-Systems
  - (k,l)-system
    - left (right) context is a word of length k(l)

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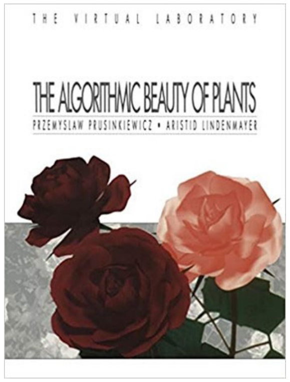
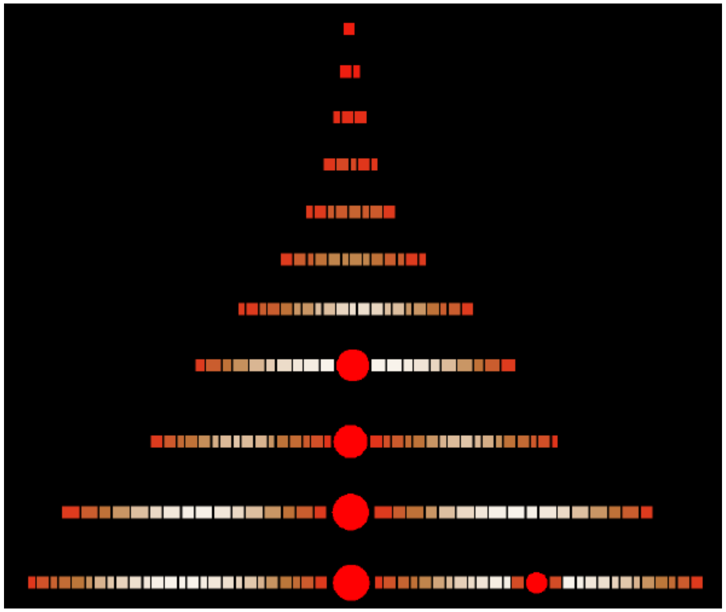
example

```
#define CH 900   /* high concentration */
#define CT 0.4   /* concentration threshold */
#define ST 3.9   /* segment size threshold */
#include H       /* heterocyst shape specification */
#ignore f ~ H

ω  :  -(90)F(0,0,CH)F(4,1,CH)F(0,0,CH)
p1 :  F(s,t,c) :  t=1 & s>=6 →
                      F(s/3*2,2,c)f(1)F(s/3,1,c)
p2 :  F(s,t,c) :  t=2 & s>=6 →
                      F(s/3,2,c)f(1)F(s/3*2,1,c)
p3 :  F(h,i,k) < F(s,t,c) > F(o,p,r) :  s>ST|c>CT →
                      F(s+.1,t,c+0.25*(k+r-3*c))
p4 :  F(h,i,k) < F(s,t,c) > F(o,p,r) :  !(s>ST|c>CT) →
                      F(0,0,CH) ~ H(1)
p5 :  H(s) :  s<3 → H(s*1.1)
```





convenient tool for expressing developmental models with ***diffusion of substances***. pattern of cells in *Anabaena catenula* and other blue-green bacteria

From: P. Prusinkiewicz and A. Lindenmayer [1991]. *The Algorithmic Beauty of Plants*.

example

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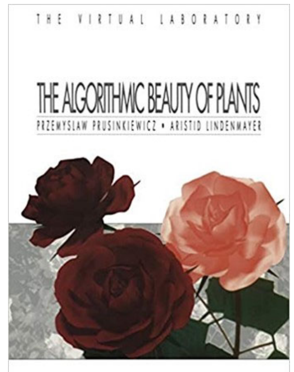
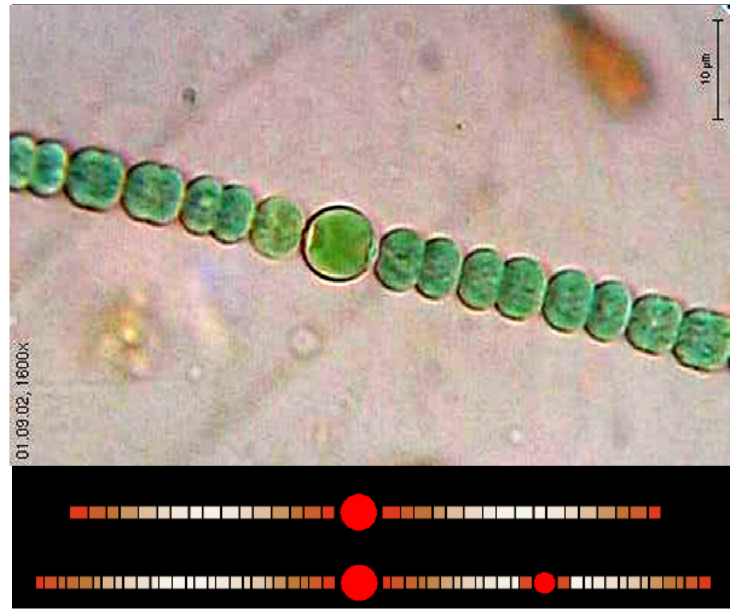
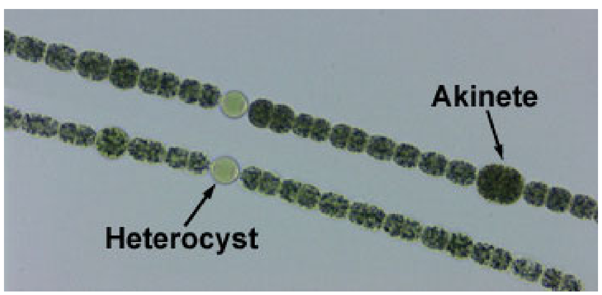
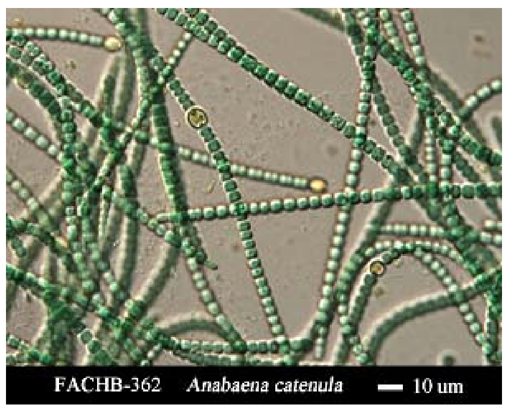
p2 :  F(s,t,c)  :  t=2 & s>=6 →
                     F(s/3,2,c)f(1)F(s/3*2,1,c)
p3 :  F(h,i,k) < F(s,t,c) > F(o,p,r)  :  s>ST|c>CT →
                     F(s+.1,t,c+0.25*(k+r-3*c))
p4 :  F(h,i,k) < F(s,t,c) > F(o,p,r)  :  !(s>ST|c>CT) →
                     F(0,0,CH) ~ H(1)
p5 :  H(s)  :  s<3 → H(s*1.1)
```





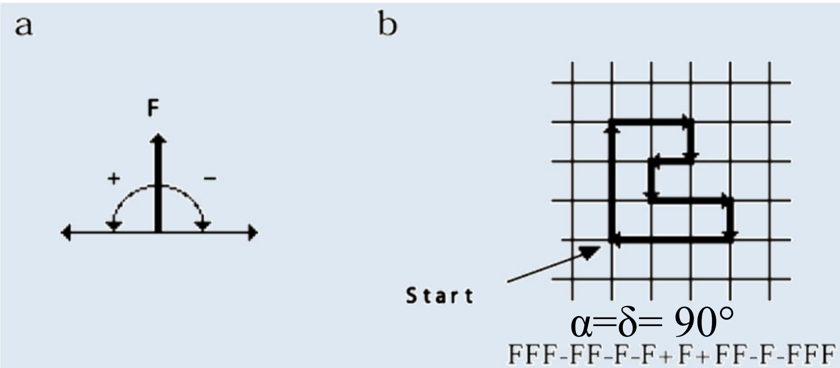




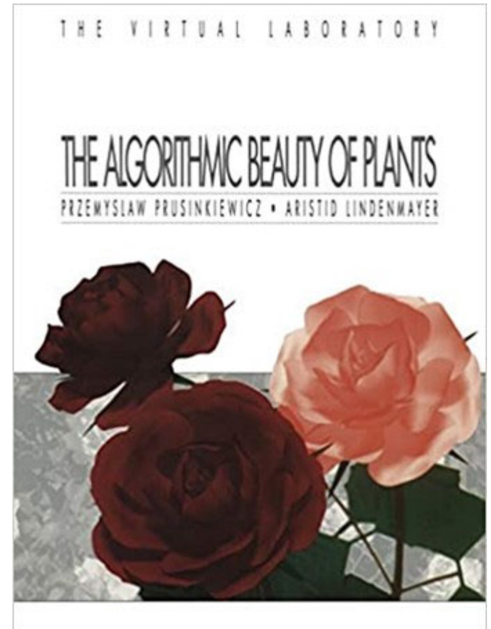
convenient tool for expressing developmental models with ***diffusion of substances***. pattern of cells in *Anabaena catenula* and other blue-green bacteria

From: P. Prusinkiewicz and A. Lindenmayer [1991]. *The Algorithmic Beauty of Plants*.

Drawing words



state of turtle defined as (x, y, α), coordinates (*position*) and angle (*heading*). Moves according to *step size* d and *angle increment* δ



F	Move forward a step of length d. The state of the turtle changes to (x', y', α), where $x' = x + d\cos\alpha$ and $y' = y + d\sin\alpha$. A line segment between points (x, y) and (x', y') is drawn.
f	Move forward a step of length d without drawing a line.
+	Turn left by angle δ. The next state of the turtle is $(x, y, \alpha+\delta)$. The positive orientation of angles is counter-clockwise.
−	Turn right by angle δ. The next state of the turtle is $(x, y, \alpha - \delta)$.

From: P. Prusinkiewicz and A. Lindenmayer [1991]. *The Algorithmic Beauty of Plants*.

alphabet handling by Turtle

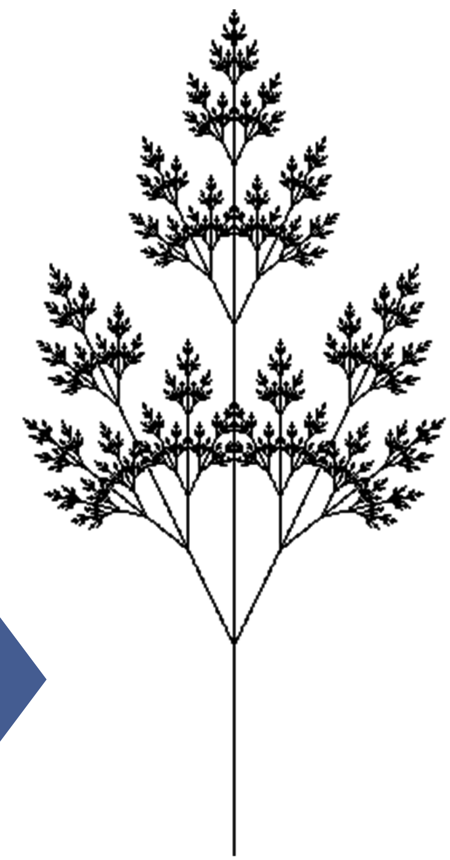
- An L-system is an ordered triplet
  - $G = <V, w, P>$
    - V = alphabet of the symbols in the system
      - V = {F, X}
    - w = nonempty word
      - the axiom: X
    - P = finite set of production rules (productions)
      - X → F[+X][-X]FX
      - F → FF

Alphabet V
{X,F,[,],+,-}

→

Drawing Procedure (Turtle)
{X,F,[,],+,-}

→



Angle: 14

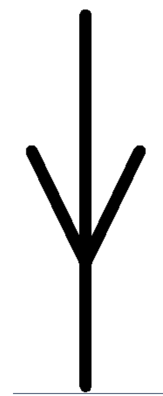
alphabet handling by Turtle

- **Example L-System**
  - V = {F, X}
  - axiom: X
  - Productions
    - X → F[+X][-X]FX
    - F → FF

Alphabet *V*
{X,F,[,],+,-}

Turtle
{X,F,[,],+,-}

n=1
F[+X][-X]FX
F[+][-]F
FF



n=2
FF[+F[+X][-X]FX][-F[+X][-X]FX]FFF[+X][-X]FX
FF[+F[+][-]F][-F[+][-]F]FFF[+][-]F
FF[+FF][-FF]FFFF

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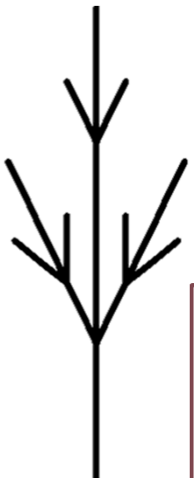
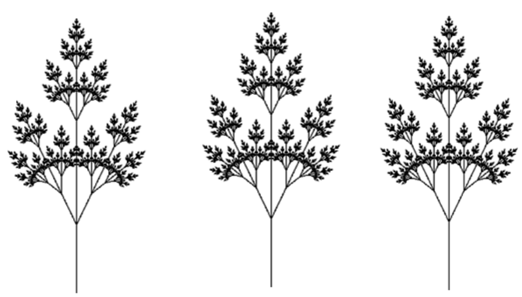
alphabet handling by Turtle



n=2
FF[+F[+X][-X]FX][-F[+X][-X]FX]FFF[+X][-X]FX
FF[+F[+][-]F][-F[+][-]F]FFF[+][-]F
FF[+FF][-FF]FFFF

n=3
FFFF[+FF[+F[+X][-X]FX][-F[+X][-X]FX]FFF[+X][-X]FX][-FF[+ F[+X][-X]FX][-F[+X][-X]FX]FFF[+X][-X]FX]FFFF[+F[+X][-X]FX][- F[+X][-X]FX]FFF[+X][-X]FX
FFFF[+FF[+F[+][-]F][- F[+][-]F]FF F[+][-]F][-FF[+ F[+][-]F][- F[+][-]F]FF F[+][-]F]FFFF[+ F[+][-]F][- F[+][-]F]FFF[+][-]F
FFFF[+FF[+FF][-FF]FFFF][-FF[+FF][-FF]FFFF]FFFF[+FF][-FF]FFFF

Angle: 14



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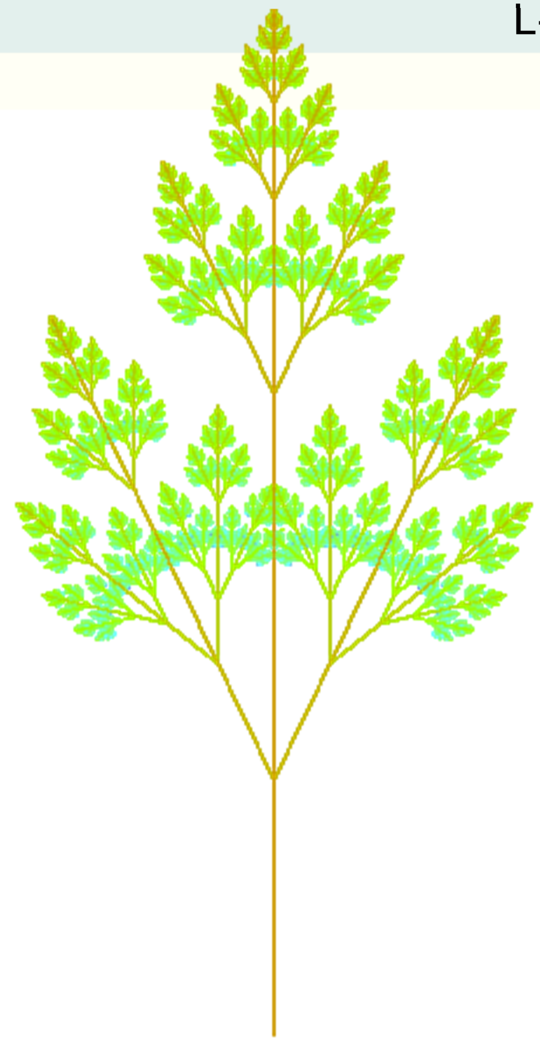
alphabet handling by Turtle (adding color)

- Example L-System
  - V = {F, X}
  - axiom: X
  - Productions
    - X → F[>8+X][>8-X]FX
    - F → FF

Alphabet *V*
{X,F,[,],+,-,>n}

Turtle
{X,F,[,],+,-,>n}



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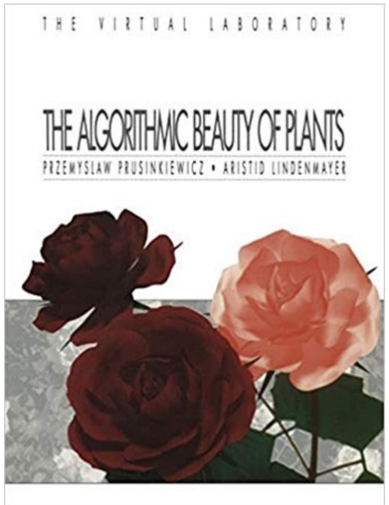
Turtle graphics

example



$$\omega: \; F-F-F-F$$
$$p: \; F \rightarrow F-F+F+FF-F-F+F$$

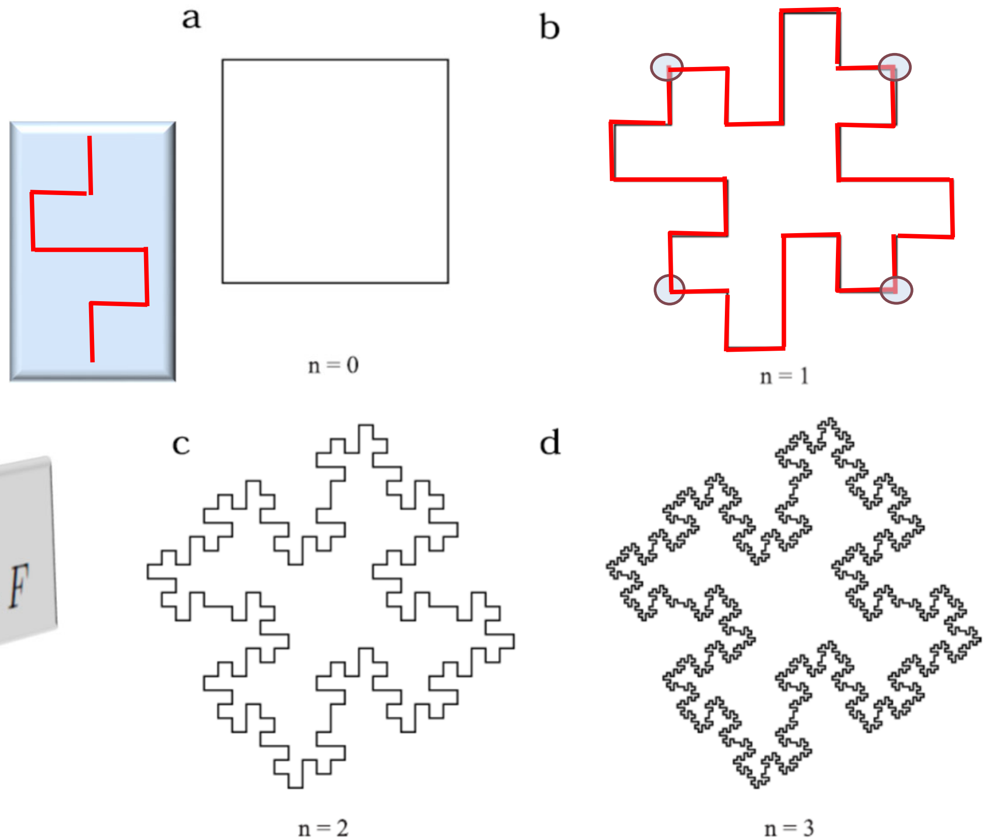
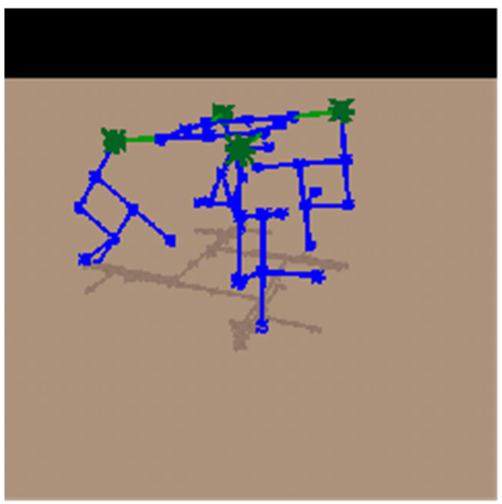


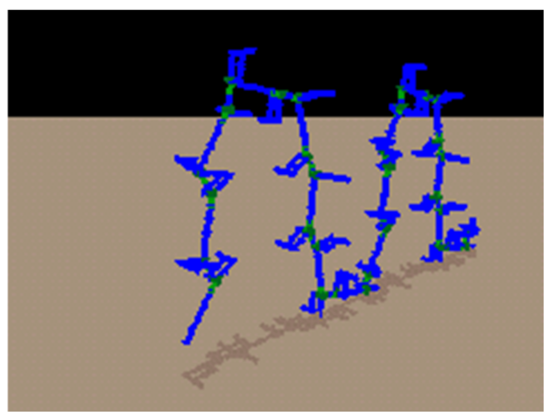
Figure 1.6: Generating a quadratic Koch island

From: P. Prusinkiewicz and A. Lindenmayer [1991]. *The Algorithmic Beauty of Plants*.

robots

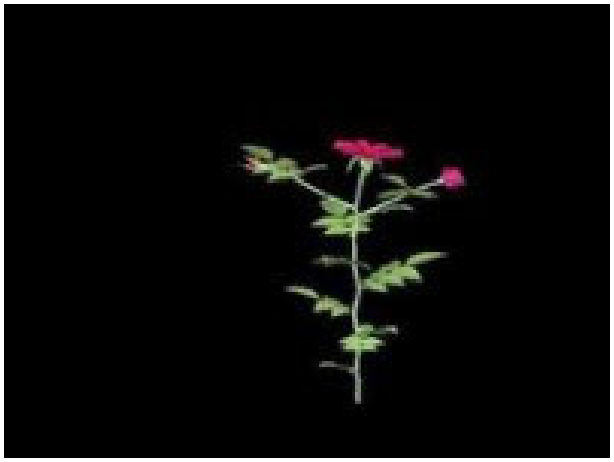
- Evolutionary design of robots
  - Difficult to reach high complexities necessary for practical engineering
  - Karl Sims and Jordan Pollack, Hod Lipson, Gregory Hornby, and Pablo Funes claim that for automatic design to scale in complexity it must employ re-used modules
    - Sims,K. [1994]. “Evolving Virtual Creatures”. *Proceedings of the 21st annual conference on Computer graphics and interactive techniques*, pp. 15 – 22.
    - H. Lipson and J. B. Pollack (2000), "Automatic design and Manufacture of Robotic Lifeforms", *Nature* **406**: 974-978.
  - *generative representation* to encode individuals in the population.
  - Indirect representation: an algorithm for creating a design.
    - using Lindenmayer systems (L-systems)
    - evolved locomotiong robots (called *genobots*).

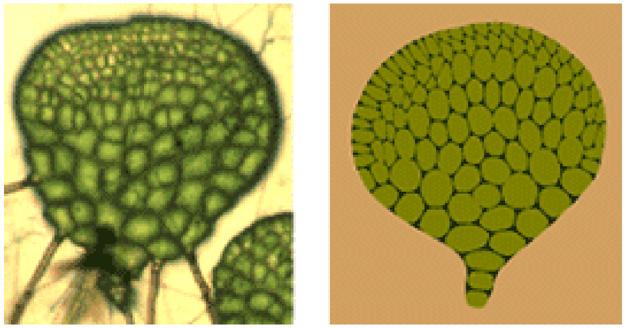




models or realistic imitations?

- Common features (design principle) between artificial and real plants
  - Development of (macro-level) morphology from local (micro-level) logic
  - Parallel application of simple rules
    - Genetic vs. algorithmic
  - Recursion
- But are the algorithms the same as the biological *mechanism*?
  - Real organisms need to economize information for coding complex phenotypes
    - The genome cannot encode every ripple of the brain or lungs
    - Organisms need to encode **compact procedures** for producing the same pattern (with randomness) again and again
- But recursion alone does not explain form and morphogenesis
  - One of the design principles involved
  - There are others
    - Selection, genetic variation, self-organization, epigenetics





fern gametophyte Microsorium linguaeforme (left) and a simulated model using map L systems (right).

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readings

- Class Book
  - Floreano, D. and C. Mattiussi [2008]. *Bio-Inspired Artificial Intelligence: Theories, Methods, and Technologies*. MIT Press.
    - **Chapter 2**.
- Lecture notes
  - Chapter 1: What is Life?
  - Chapter 2: The logical Mechanisms of Life
  - Chapter 3: Formalizing and Modeling the World
    - posted online @ http://informatics.indiana.edu/rocha/i-bic

- Papers and other materials
  - Optional
    - Nunes de Castro, Leandro [2006]. *Fundamentals of Natural Computing: Basic Concepts, Algorithms, and Applications*. Chapman & Hall.
      - Chapter 2, all sections
      - Chapter 7, sections 7.3 – Cellular Automata
      - Chapter 8, sections 8.1, 8.2, 8.3.10
    - Flake's [1998], *The Computational Beauty of Life*. MIT Press.
      - Chapters 10, 11, 14 – Dynamics, Attractors and chaos
    - Prusinkiewicz and Lindenmeyer [1996] *The algorithmic beauty of plants*.
      - Chapter 1